



## Amazing answer of ChatGPT to my maths question (Hahn-Banach without Zorn ?)

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**FO:** Does there exist a constructive proof of the Hahn-Banach Theorem without Zorn's Lemma?

**ChatGPT:** No, the Hahn-Banach theorem cannot be proven constructively without some form of the Axiom of Choice, which includes Zorn's Lemma. In fact, the Hahn-Banach theorem is equivalent to the ultrafilter lemma, which is a weaker form of the Axiom of Choice.

The ultrafilter lemma states that for any set  $X$ , there exists an ultrafilter on  $X$ . An ultrafilter is a collection of subsets of  $X$  that satisfies certain properties, including being closed under supersets, non-empty, and not containing both a set and its complement.

The Hahn-Banach theorem is a powerful result in functional analysis that asserts the existence of certain kinds of functionals on a vector space. The proof of this theorem relies on the Zorn's Lemma, which is equivalent to the Axiom of Choice. While there are some weaker versions of the Axiom of Choice that can be used to prove the Hahn-Banach theorem, they still require some form of choice.

Therefore, any proof of the Hahn-Banach theorem without Zorn's Lemma or some form of the Axiom of Choice would be considered non-constructive.