Bilateral first-to-default CVA and Basel III-CVA risk charges embedded in the context of an integrated portfolio risk model – First thoughts

Non-Confidential Version

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This is work in progress. In particular, definitions, abbreviations and symbolic language in this work can be subject of change (ambiguity of terms in the literature).

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On CCR I

Suppose there are two parties who are trading a portfolio of OTC derivative contracts such as, e. g., a portfolio of CDSs. (Bilateral) counterparty credit risk (CCR) is the risk that *at least one* of those two parties in that derivative transaction will default prior to the expiration of the contract and will be unable to make all contractual payments to its counterpart.

A derivative's underlying in general depends on further parties which might default (partially or even altogether) before the final settlement of the transaction's cash flows. $\Rightarrow$ "infectious defaults" respectively multivariate dependence of random default times and market risk factors.

On CCR II

- CCR is of bilateral nature, based on a contractual exchange of cashflows between two parties over a period of time.
- Future cashflow exchanges are not known with certainty today. The main feature that distinguishes CCR from the risk of a standard loan is the uncertainty of the exposure at any future date. $\Rightarrow$ Regarding the modelling of exposure a simulation of future cashflow exchanges is necessary (MC, AMC, SDEs, PDEs, SPDEs, grid computing . . .).
- Wrong-Way Risk (WWR): Strong relationship between credit risk and market risk (dependence between default time and MtM value at default time). $\Rightarrow$ We need a truly dynamic (portfolio) credit risk model for both parties: static copula models are not enough. Default intensities should depend on economic factors.

Formalisation of CCR I

Let us consider an investor (Alice) and its counterparty (Bob), trading a portfolio of N derivative contracts, expiring completely at final maturity $T > 0$. Fix an arbitrary contract of this portfolio, contract i, say. Assume that Bob will default before Alice and before T. Firstly, let us assume that neither Alice nor Bob receives or posts collateral until T.

The contract's market value $V^{(i)}_{\text{Alice}}(t;T)$ viewed from the perspective of Alice is known at $t = 0$ only. For any other t, the value $V^{(i)}_{\text{Alice}}(t;T)$ is unknown to all agents in the market, hence a random variable.

Since Bob defaults at time $\tau_{\text{Bob}} \leq T$, the trade has to be *closed-out* at τ_{Bob} by Alice (who survives Bob) at the random(!) market value $V^{(i)}_{\text{Alice}}(\tau_{\text{Bob}};T)$, and it has to be replaced. The close-out practice of Alice proceeds as follows:

Formalisation of CCR II

- If $V_{\text{Alice}}^{(i)}(\tau_{\text{Bob}};T) > 0$, Alice receives $R_{\text{Bob}}^{(i)} \cdot V_{\text{Alice}}^{(i)}(\tau_{\text{Bob}};T) > 0$ from Bob, where $0 < R_{\text{Bob}}^{(i)} \leq 1$ is Bob's recovery rate. Yet she has to pay $V_{\text{Alice}}^{(i)}(\tau_{\text{Bob}};T)$ to a third party in order to replace the contract. Thus, Alice's net loss is given by $\text{LGD}_{\text{Bob}}^{(i)} \cdot V_{\text{Alice}}^{(i)}(\tau_{\text{Bob}};T)$, where $\text{LGD}_{\text{Bob}}^{(i)} := 1 - R_{\text{Bob}}^{(i)}$.
- If $V_{\text{Alice}}^{(i)}(\tau_{\text{Bob}};T) \leq 0$ Alice receives $-V_{\text{Alice}}^{(i)}(\tau_{\text{Bob}},T)$ from a further and replacing counterparty but she has to forward this amount to Bob. Thus, Alice's net loss $= 0$.

Thus, given that Bob defaults before Alice and before T, at τ_{Bob}, the amount of Alice's loss equals $\text{LGD}_{\text{Bob}}^{(i)} \cdot (V_{\text{Alice}}^{(i)}(\tau_{\text{Bob}};T))^{+}$, where $x^{+} := \max\{x,0\}$. At t the number $(V_{\text{Alice}}^{(i)}(t;T))^{+}$ is her current exposure to Bob (aka as replacement cost) with respect to contract i at t.

Formalisation of CCR III

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Note that we did not specify the random structure of $\mathrm{LGD}_{\mathrm{Bob}}^{(i)}$. $\mathrm{LGD}_{\mathrm{Bob}}^{(i)}$ might depend on τ_{Bob} as well.

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Natural Question (. . . DVA?)

What would be Alice's (net) loss if she defaulted before Bob and before T?

Formalisation of CCR III

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Natural Question (. . . DVA?)

What would be Alice's (net) loss if she defaulted before Bob and before T?

Answer

Actually, Alice would not loose. It's namely Bob who would loose: **swap** *the roles of Alice and Bob! Consequently, if Bob's loss is the negative of Alice's gain (money conservation) Alice would gain the following positive amount:*

$$\mathit{LGD}^{(i)}_{\mathit{Alice}} \cdot (V^{(i)}_{\mathit{Bob}}(\tau_{\mathit{Alice}}; T))^{+} = \mathit{LGD}^{(i)}_{\mathit{Alice}} \cdot (V^{(i)}_{\mathit{Alice}}(\tau_{\mathit{Alice}}; T))^{-},$$

where $x^{-} := x^{+} - x = (-x)^{+}$.

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Netting agreements I

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Some of Alice's derivative trades with Bob could offset each other. If all trades $i = 1, \ldots, N$ were settled separately at t, Alice's total loss amount would be given as $\sum_{i=1}^{N} \mathrm{LGD}_{\mathrm{Bob}}^{(i)}(V_{\mathrm{Alice}}^{(i)}(\tau_{\mathrm{Bob}}; T))^{+}$, implying that even completely offsetting trades could generate a strictly positive total loss amount - such as the following two ones:

$$1/3 \cdot 100 + 1/3 \cdot (-100) = 0 < 1/3 \cdot 100 + 0 = 1/3 \cdot 100^{+} + 1/3 \cdot (-100)^{+}.$$

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$$1/3{\cdot}100+1/3{\cdot}(-100) = 0 < 1/3{\cdot}100+0 = 1/3{\cdot}100^{+}+1/3{\cdot}(-100)^{+}.$$

Retrospection

A netting agreement is a legally binding agreement between two parties stipulating that if a party defaults, legal obligations arising from derivative transactions covered by this agreement must be based solely on the net value of such transactions. A set consisting of all non-single trades under a single netting agreement or a single (non-nettable) trade is called netting set.

Netting agreements II

Consequently, in each netting set derivatives with positive value at the time of default offset the ones with negative value (at least partially).

Suppose that the portfolio of derivatives of Alice is a disjoint union of K netting sets $\mathcal{N}_k$, $k = 1, \dots, K$. Per netting set $\mathcal{N}_k$ Alice's total loss amount would be given as $\text{LGD}^{(k)}_{\text{Bob}} \cdot \left(\sum_{i \in \mathcal{N}_k} V^{(i)}_{\text{Alice}}(\tau_{\text{Bob}}; T) \right)^+$. Thus, her total loss amount is given by

$$\sum_{k=1}^{K} \text{LGD}^{(k)}_{\text{Bob}} \cdot \left(\sum_{i \in \mathcal{N}_k} V^{(i)}_{\text{Alice}}(\tau_{\text{Bob}}; T) \right)^+ .$$

Consequently, the exposure at portfolio level is simply the sum of the exposures of the sub-portfolios in the netting sets. Thus, WLOG we may restrict our investigation to a single netting set ($K := 1$), consisting of a single derivative contract.

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Bilateral CCR in a financial network

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Consider the following 3 parties (embedded in a financial network, consisting of a finite number of trading parties as nodes): an *investor* (Alice), a *reference credit*, and the *investor's counterpart* (Bob). Let us number them by 0, 1, and 2 (where 1 subscripts the reference credit). Let τ_k denote the random default time of party k and T the final maturity of the payoff of the traded portfolio of derivatives. Fix $k \in \{0, 2\}$. Put $N := \{\tau_0 > T \text{ and } \tau_2 > T\}$ and $G_k := \{\tau_k \leq T\}$. Then G_k can be written as the disjoint union of the sets $A_k := \{\tau_k \leq T \text{ and } \tau_k = \tau_{2-k}\} = A_{2-k}$ (simultaneous default before T) and $\Omega \setminus A_k = A_k^- \dot\cup A_k^+$, where $A_k^- := \{\tau_k \leq T \text{ and } \tau_k < \tau_{2-k}\}$ and $A_k^+ := \{\tau_k \leq T \text{ and } \tau_k > \tau_{2-k}\}$. Note that $A_k^+ \subseteq A_{2-k}^-$. Hence,

$$\Omega = N \dot\cup G_0 \dot\cup G_2 \stackrel{\checkmark}{=} N \dot\cup A_0^- \dot\cup A_2^- \dot\cup A_0 .$$

Money conservation

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Definition (Money Conservation Principle)

Let x be an arbitrary amount of money (which could be a negative number), measured in a single fixed currency unit $\mathbb{U}$ (e.g., $\mathbb{U}$: = €). Let $k \in \{0, 2\}$. TFAE:

- Party k receives x currency units $\mathbb{U}$ from party $2 - k$;
- Party $2 - k$ pays x currency units $\mathbb{U}$ to party k;
- Party $2 - k$ receives $-x$ currency units $\mathbb{U}$ from party k.

Thus, paying x currency units $\mathbb{U}$ is by definition equivalent to receiving $-x$ currency units $\mathbb{U}$ for all real money values x (a loss (resp. liability) is a negative gain (resp. asset)). Consequently, for any point in time $0 \leq t \leq T$ any cash flow Π_t to party 2 (an asset if $\Pi_t \geq 0$), is precisely the cash flow $-\Pi_t$ to party 0 (a liability if $\Pi_t \geq 0$).

The role of partial information

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Fix $0 \leq t < u \leq T$ and $k \in \{0, 2\}$. Let $\mathcal{F}_t$ denote the information of a specific investor at t, representing all observable market quantities but the default events or any factors that might be linked to credit ratings of the both parties, and let $\mathcal{G}_t$ represent the investor's enlarged information at time t, consisting of knowledge of the behaviour of market prices up to time t as well as (possible) default times until t. With respect to the information $\mathcal{F}_t$ defaults until t would arrive suddenly, as opposed to the case of the enlarged information $\mathcal{G}_t$.

Throughout this presentation, the letter ω always describes a random "event". We consider functions of type $\mathbb{1}_A$, where $\mathbb{1}_A(\omega) := 1$ if $\omega \in A$ and $\mathbb{1}_A(\omega) := 0$ if $\omega \notin A$. Moreover, CCR analysis is based on the functions $x^+ := \max\{x, 0\}$ and $x^- := x^+ - x = (-x)^+ = \max\{-x, 0\}$. We now introduce the following very important notation:

The main CCR building blocks

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$\Pi_k^{(t,u]} \overset{(!)}{=} -\Pi_{2-k}^{(t,u]}$

Party k's received random CCR clean net cash flow from the claim in $(t,u]$, discounted to time t

The main CCR building blocks

$$\Pi_k^{(t,u]} \overset{(!)}{=} -\Pi_{2-k}^{(t,u]}$$

Party k's received random CCR clean net cash flow from the claim in $(t,u]$, discounted to time t

$$V_k(t;T) := \mathbb{E}_{\mathbb{Q}}[\Pi_k^{(t,T]} | \mathcal{G}_t] = -V_{2-k}(t;T)$$

Random NPV (or MtM) of $\Pi_k^{(t,u]}$ given as conditional expectation w.r.t. a risk neutral measure $\mathbb{Q}$, given the information $\mathcal{G}_t$ (cf. [2])

The main CCR building blocks

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Random NPV (or MtM) of $\Pi_k^{(t,u]}$ given as conditional expectation w.r.t. a risk neutral measure $\mathbb{Q}$, given the information $\mathcal{G}_t$ (cf. [2])

$$0 \leq R_k < 1$$

k's (rdm.) recovery rate; i. e., the portion of the payoff from the MtM paid by party k to party $2-k$ in case of k's default

The main CCR building blocks

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$\Pi_k^{(t,u]} \overset{(!)}{=} -\Pi_{2-k}^{(t,u]}$	Party k's received random CCR clean net cash flow from the claim in $(t,u]$, discounted to time t
$V_k(t;T) := \mathbb{E}_{\mathbb{Q}}[\Pi_k^{(t,T]} \mid \mathcal{G}_t] = -V_{2-k}(t;T)$	Random NPV (or MtM) of $\Pi_k^{(t,u]}$ given as conditional expectation w.r.t. a risk neutral measure $\mathbb{Q}$, given the information $\mathcal{G}_t$ (cf. [2])
$0 \leq R_k < 1$	k's (rdm.) recovery rate; i. e., the portion of the payoff from the MtM paid by party k to party $2-k$ in case of k's default
$0 < \text{LGD}_k := 1 - R_k \leq 1$	k's (random) Loss Given Default
$D(t,u) = \exp\left(-\int_t^u r(s)ds\right)$	random discount factor at time t for time u

Example – Part 1

Example (One-Period CDS)

Let $r, c, T > 0$*. Put* $k := 0$*,* $t := 0$*, and let* $D(0,T) = e^{-rT}$*. Assume that* $\tau_0 = +\infty$ *(i. e., no default of party* 0*). Let* τ_1 *be the default time of a third party (a reference name). Assume that* $0 < q_1 := \mathbb{Q}(\tau_1 \leq T) < 1$*. Party* 0 *is the protection buyer of the following simple CDS with notional* 1*, sold by party* 2*. If party* 2 *does not default until* T*, party* 0 *will receive the following net cash flow:*

$$\Pi_0^{(0,T]}(\omega) := \begin{cases} -c \cdot e^{-rT}, & \text{if } \tau_1(\omega) > T \\ 1 \cdot e^{-rT} & \text{if } \tau_1(\omega) \leq T \end{cases}$$

Thus, $V_0(0;T) = \mathbb{E}_{\mathbb{Q}}\left[\Pi_0^{(0,T]}\right] = e^{-rT}(-c(1-q_1) + q_1) = e^{-rT}(-c + q_1(1+c))$*.*

Example – Part 2

Problem

Given the assumption that party 2 could default before T, how would then party 0's net cashflow change if party 2 had to pay $\frac{1}{3}$ at T if the reference name defaulted before T, and if party 2 defaulted before the reference name? Assume further that if the reference name defaulted after T, the protection buyer still would have to pay c.

Example – Part 2

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Problem

Given the assumption that party 2 could default before T, how would then party 0's net cashflow change if party 2 had to pay $\frac{1}{3}$ at T if the reference name defaulted before T, and if party 2 defaulted before the reference name? Assume further that if the reference name defaulted after T, the protection buyer still would have to pay c.

What would be a "fair price" which should be paid by party 2 to the protection buyer (i. e., to party 0) at $t = 0$ to compensate for a possible default of protection?

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ISDA's Full Two-Way Payment Rule

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Fix $k \in \{0,2\}$, and assume that party $2-k$ defaults first. Suppose that close-out is settled at τ_{2-k}. According to the *Full Two-Way Payment Rule* under ISDA Master Agreements (in a given netting set) we obtain the following table:

	$V_k(\tau_{2-k};T) > 0$	$V_k(\tau_{2-k};T) \leq 0$
Party k receives from party $2-k$	$R_{2-k} \cdot V_k(\tau_{2-k};T)$	0
Party k pays to party $2-k$	0	$-V_k(\tau_{2-k};T)$

In the following, given that $k \in \{0,2\}$ and $0 \leq t \leq u < T$ (T fixed),

$$U_k(t,u) := D(t,u)V_k(u;T)$$

will denote party k's received MtM at u, discounted back to t.

Vulnerable cash flows I

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Definition (risk-free close-out cash flow - according to ISDA)

Let $k \in \{0, 2\}$ and $0 \leq t \leq \min\{\tau_0, \tau_2\}$. If party $2-k$ defaults first, the risk-free (or CCR clean) close-out cash flow seen from the perspective of party $2-k$ at τ_{2-k} is given by $-R_{2-k}(V_k(\tau_{2-k}; T))^+ + (V_k(\tau_{2-k}; T))^-$ and hence by $-R_{2-k}(U_{2-k}(t, \tau_{2-k}))^- + (U_{2-k}(t, \tau_{2-k}))^+$ at t.

Thus, a strict use of the Money Conservation Principle (MCP) leads to the important

Proposition

Let $k, l \in \{0, 2\}$ and $0 \leq t \leq \min\{\tau_0, \tau_2\}$. If party l defaults first, the risk-free close-out cash flow seen from the perspective of party k at t is given by

$$(-1)^{\frac{k+l}{2}} \cdot \Big(LGD_l \cdot (U_l(t, \tau_l))^- + U_l(t, \tau_l) \Big).$$

Vulnerable cash flows II

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In the following, let us assume that $0 \leq t < \min\{\tau_0, \tau_2\}$ (on Ω). Fix $k \in \{0, 2\}$, and let $\widehat{\Pi}_k^{(t,T]}$ denote party k's discounted received payoff of a generic defaultable claim at t. The subsequently following construction of $\widehat{\Pi}_k^{(t,T]}$ is built on first-to-default scenarios. We assume the validness of the Money Conservation Principle, and we are going to implement the risk-free Full Two-Way Payment Rule under ISDA Master Agreements. Let us further assume that there are (almost) no simultaneous defaults. Hence, we may put $\mathbb{1}_{A_0} = \mathbb{1}_{A_2} = 0$. Consequently, due to the previous Proposition it immediately follows that

Vulnerable cash flows III

$$\widehat{\Pi}_k^{(t,T]} = 1\!\!1_N \cdot \Pi_k^{(t,T]} + 1\!\!1_{A_{2-k}^-} \cdot \Pi_k^{(2-k)} + 1\!\!1_{A_k^-} \cdot \Pi_k^{(k)}, \qquad (1)$$

where the 2×2 random matrix $(\Pi_k^{(l)})_{l,k \in \{0,2\}}$ is given by

$$\Pi_k^{(l)} := \Pi_k^{(t,\tau_l]} + (-1)^{\frac{k+l}{2}} \cdot \Big(\mathsf{LGD}_l \cdot (U_l(t,\tau_l))^- + U_l(t,\tau_l)\Big)$$

for all $l \in \{k, 2-k\}$. Observe that $\widehat{\Pi}_k^{(t,T]} \overset{\checkmark}{=} -\widehat{\Pi}_{2-k}^{(t,T]}$ (i.e., the MCP carries over to the “CCR cash flows”). Moreover, $1\!\!1_N \cdot \widehat{\Pi}_k^{(t,T]} = 1\!\!1_N \cdot \Pi_k^{(t,T]}$ and $1\!\!1_{A_l^-} \cdot \widehat{\Pi}_k^{(t,T]} = 1\!\!1_{A_l^-} \cdot \Pi_k^{(l)}$ for all $l \in \{k, 2-k\}$ (due to equation (1)). To uncover the general structure of CVA and DVA, we have to take a closer look at the both random variables $\Pi_k^{(k)}$ and $\Pi_k^{(2-k)}$.

Vulnerable cash flows IV

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Lemma

Let $0 \leq s < t < u < T$, $k \in \{0, 2\}$ *and assume that there are no arbitrage opportunities. Then*

(i) $\Pi_k^{(s,t]} + D(s,t) \cdot \Pi_k^{(t,u]} = \Pi_k^{(s,u]}$.

(ii) $V_k(s;u) - V_k(s;t) = \mathbb{E}_{\mathbb{Q}}[D(s,t) \cdot V_k(t;u) | \mathcal{G}_s]$.

Proof.

Exercise. Hint: $\mathbb{E}_s \mathbb{E}_t = \mathbb{E}_s$, where $\mathbb{E}_x := \mathbb{E}_{\mathbb{Q}}[\cdot | \mathcal{G}_x]$. □

Vulnerable cash flows V

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Firstly note that

$$(-1)^{\frac{k+l}{2}} \cdot U_l(t,\tau_l) \overset{(!)}{=} U_k(t,\tau_l) \qquad (2)$$

for all $l \in \{k, 2-k\}$ (due to a second application of the MCP). Hence,

$$\begin{aligned} \Pi_k^{(l)} &= \Pi_k^{(t,\tau_l]} + (-1)^{\frac{k+l}{2}} \cdot U_l(t,\tau_l) + (-1)^{\frac{k+l}{2}} \cdot \text{LGD}_l \cdot (U_l(t,\tau_l))^- \\ &\overset{(2)}{=} \Pi_k^{(t,\tau_l]} + U_k(t,\tau_l) + (-1)^{\frac{k+l}{2}} \cdot \text{LGD}_l \cdot (U_l(t,\tau_l))^- \end{aligned}$$

Consequently, due to the above Lemma and the definition of MtM, an application of the conditional expectation operator $\mathbb{E}_{\mathbb{Q}}[\cdot|\mathcal{G}_t]$ to $\mathbb{1}_{A_l^-}\Pi_k^{(l)}$ further implies that

$$\begin{aligned}\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\Pi_k^{(l)}|\mathcal{G}_t\right] &= \mathbb{1}_{A_l^-}\mathbb{E}_{\mathbb{Q}}\left[\Pi_k^{(l)}|\mathcal{G}_t\right] \\ &= \mathbb{1}_{A_l^-}V_k(t;\tau_l) \\ &+ \mathbb{1}_{A_l^-}\left(V_k(t;T)-V_k(t;\tau_l)\right) \\ &+ (-1)^{\frac{k+l}{2}}\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\mathrm{LGD}_l(U_l(t,\tau_l))^-|\mathcal{G}_t\right]\end{aligned}$$

for all $l \in \{k, 2-k\}$. In particular,

FtDCVA I

Consequently, due to the above Lemma and the definition of MtM, an application of the conditional expectation operator $\mathbb{E}_{\mathbb{Q}}[\cdot|\mathcal{G}_t]$ to $\mathbb{1}_{A_l^-}\Pi_k^{(l)}$ further implies that

$$\begin{aligned}\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\Pi_k^{(l)}\middle|\mathcal{G}_t\right] &= \mathbb{1}_{A_l^-}\mathbb{E}_{\mathbb{Q}}\left[\Pi_k^{(l)}\middle|\mathcal{G}_t\right] \\ &= \mathbb{1}_{A_l^-}V_k(t;\tau_l) \\ &+ \mathbb{1}_{A_l^-}\big(V_k(t;T)-V_k(t;\tau_l)\big) \\ &+ (-1)^{\frac{k+l}{2}}\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\mathsf{LGD}_l(U_l(t,\tau_l))^-\middle|\mathcal{G}_t\right]\end{aligned}$$

for all $l \in \{k, 2-k\}$. In particular,

$$\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\widehat{\Pi}_k^{(t,T]}\middle|\mathcal{G}_t\right] = \mathbb{1}_{A_l^-}V_k(t;T) + (-1)^{\frac{k+l}{2}}\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\mathsf{LGD}_l(U_l(t,\tau_l))^-\middle|\mathcal{G}_t\right]$$

FtDCVA I

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Consequently, due to the above Lemma and the definition of MtM, an application of the conditional expectation operator $\mathbb{E}_{\mathbb{Q}}[\cdot|\mathcal{G}_t]$ to $\mathbb{1}_{A_l^-}\Pi_k^{(l)}$ further implies that

$$\begin{aligned}
\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\Pi_k^{(l)}\middle|\mathcal{G}_t\right] &= \mathbb{1}_{A_l^-}\mathbb{E}_{\mathbb{Q}}\left[\Pi_k^{(l)}\middle|\mathcal{G}_t\right] \\
&= \mathbb{1}_{A_l^-}V_k(t;\tau_l) \\
&+ \mathbb{1}_{A_l^-}\left(V_k(t;T)-V_k(t;\tau_l)\right) \\
&+ (-1)^{\frac{k+l}{2}}\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\mathsf{LGD}_l(U_l(t,\tau_l))^-\middle|\mathcal{G}_t\right]
\end{aligned}$$

for all $l \in \{k, 2-k\}$. In particular,

$$\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\widehat{\Pi}_k^{(t,T]}\middle|\mathcal{G}_t\right] = \mathbb{1}_{A_l^-}V_k(t;T) + (-1)^{\frac{k+l}{2}}\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\mathsf{LGD}_l(U_l(t,\tau_l))^-\middle|\mathcal{G}_t\right]$$

for all $l \in \{k, 2-k\}$.

Consequently, since $1 = \mathbb{1}_N + \mathbb{1}_{A^-_{2-k}} + \mathbb{1}_{A^-_k}$, we finally obtain

$$\begin{aligned}\mathbb{E}_{\mathbb{Q}}\left[\widehat{\Pi}_k^{(t,T]}|\mathcal{G}_t\right] &= V_k(t;T) \\ &- \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A^-_{2-k}}\mathrm{LGD}_{2-k}(U_{2-k}(t,\tau_{2-k}))^-\big|\mathcal{G}_t\right] \\ &+ \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A^-_k}\mathrm{LGD}_k(U_k(t,\tau_k))^-\big|\mathcal{G}_t\right],\end{aligned}$$

Since $\mathcal{F}_t \subseteq \mathcal{G}_t$ and $V_k(t;T) = \mathbb{E}_{\mathbb{Q}}\left[\Pi_k^{(t,T]}|\mathcal{G}_t\right]$, the tower property of the conditional expectation operator finally implies that

FtDCVA III

$$
\begin{aligned}
\mathbb{E}_{\mathbb{Q}}\left[\widehat{\Pi}_k^{(t,T]}|\mathcal{F}_t\right] &= \mathbb{E}_{\mathbb{Q}}\left[\Pi_k^{(t,T]}|\mathcal{F}_t\right] \\
&- \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_{2-k}^-}\mathsf{LGD}_{2-k}(U_{2-k}(t,\tau_{2-k}))^-\Big|\mathcal{F}_t\right] \\
&+ \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_k^-}\mathsf{LGD}_k(U_k(t,\tau_k))^-\Big|\mathcal{F}_t\right].
\end{aligned}
$$

Put

$$
\begin{aligned}
\mathsf{FtDCVA}_k(t;T) &:= \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_{2-k}^-}\mathsf{LGD}_{2-k}(U_{2-k}(t,\tau_{2-k}))^-\Big|\mathcal{F}_t\right] \\
&= \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_{2-k}^-}\mathsf{LGD}_{2-k}D(t,\tau_{2-k})(V_k(\tau_{2-k};T))^+\Big|\mathcal{F}_t\right]
\end{aligned}
$$

$$\mathsf{FtDDVA}_k(t;T) := \mathsf{FtDCVA}_{2-k}(t;T) \text{ and}$$

$$\mathsf{BFtDCVA}_k(t;T) := \mathsf{FtDCVA}_k(t;T) - \mathsf{FtDDVA}_k(t;T).$$

Definition

Let $k = 0$ or $k = 2$. Let $\mathbb{Q}$ be a risk neutral probability measure and $\mathcal{F}_t$ represent the information about all observable market quantities but the default events at t. Let $0 \leq t < \min\{\tau_0, \tau_2\}$ $\mathbb{Q}-$a.s..

(i) The positive $\mathcal{F}_t$-measurable random variable $\text{FtDCVA}_k(t; T)$ is called First-to-Default Credit Valuation Adjustment at t, paid by party $2-k$ to party k.

(ii) $0 \leq \text{FtDDVA}_k(t; T) := \text{FtDCVA}_{2-k}(t; T)$ is called First-to-Default Debit Credit Valuation Adjustment at t, paid by party k to party $2-k$.

(iii) $\text{BFtDCVA}_k(t; T) := \text{FtDCVA}_k(t; T) - \text{FtDDVA}_k(t; T)$ is called Bilateral First-to-Default Credit Valuation Adjustment at t, viewed from party k's perspective.

FtDCVA V

Observe that in general it is not possible to position $\mathbb{1}_{A_l^-}$ in front of the conditional expectation operator $\mathbb{E}_{\mathbb{Q}}\left[\,\cdot\,\middle|\mathcal{F}_t\right]$! Given the observable market quantities but the default events $\mathcal{F}_t$ only, a possible default of party l is not encoded in $\mathcal{F}_t$.

So, we have arrived at the following crucial result (cf. [4]):

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Theorem (Brigo-Capponi)

Let $\mathbb{Q}$ *be a "risk neutral" probability measure. Let* $0 \leq t < \min\{\tau_0, \tau_2\}$ $\mathbb{Q} -$ *a.s. and* $\mathcal{F}_t$ *denote the information at* t*, representing all the observable market quantities but the default events. Assume that the MCP holds. Under ISDA's risk-free Full Two-Way Payment Rule it follows that*

FtDCVA VI

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$$\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\widehat{\Pi}_k^{(t,T]}\middle|\mathcal{F}_t\right] - \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\Pi_k^{(t,T]}\middle|\mathcal{F}_t\right] = (-1)^{\frac{k+l}{2}} \cdot \mathit{FtDCVA}_{2-l}(t;T)$$

for all $k, l \in \{0, 2\}$,

FtDCVA VI

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FtDCVA VI

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Let $\mathbb{Q}$ be a "risk neutral" probability measure. Let $0 \leq t < \min\{\tau_0, \tau_2\}$ $\mathbb{Q}-$ a.s. and $\mathcal{F}_t$ denote the information at t, representing all the observable market quantities but the default events. Assume that the MCP holds. Under ISDA's risk-free Full Two-Way Payment Rule it follows that

$$\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\widehat{\Pi}_k^{(t,T)}\middle|\mathcal{F}_t\right] - \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A_l^-}\Pi_k^{(t,T)}\middle|\mathcal{F}_t\right] = (-1)^{\frac{k+l}{2}} \cdot \mathit{FtDCVA}_{2-l}(t;T)$$

for all $k, l \in \{0, 2\}$, and

$$\begin{aligned}\mathbb{E}_{\mathbb{Q}}[\widehat{\Pi}_k^{(t,T)}|\mathcal{F}_t] &= \mathbb{E}_{\mathbb{Q}}[\Pi_k^{(t,T)}|\mathcal{F}_t] - \mathit{BFtDCVA}_k(t;T)\\ &= \mathbb{E}_{\mathbb{Q}}[\Pi_k^{(t,T)}|\mathcal{F}_t] - \mathit{FtDCVA}_k(t;T) + \mathit{FtDDVA}_k(t;T)\end{aligned}$$

for all $k \in \{0, 2\}$.

UCVA$_k(t;T)$ as a special case of FtDCVA$_k(t;T)$

Special Case (A single default only $\rightsquigarrow$ Basel III)

Fix $k \in \{0,2\}$. Assume that in addition $\tau_k = +\infty$ (i. e., no default of party k). Then $A^-_{2-k} = \{\tau_{2-k} \leq T\}$ and $A^-_k = \emptyset$. Consequently, FtDDVA$_k(t;T) = 0$,

$$\mathbb{E}_{\mathbb{Q}}\big[\widehat{\Pi}_k^{(t,T]}|\mathcal{F}_t\big] = \mathbb{E}_{\mathbb{Q}}\big[\Pi_k^{(t,T]}|\mathcal{F}_t\big] - \textit{FtDCVA}_k(t;T)\,,$$

$\text{UCVA}_k(t;T)$ as a special case of $\text{FtDCVA}_k(t;T)$

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and

$$\mathbb{E}_{\mathbb{Q}}\left[\widehat{\Pi}_{2-k}^{(t,T]}|\mathcal{F}_t\right] = \mathbb{E}_{\mathbb{Q}}\left[\Pi_{2-k}^{(t,T]}|\mathcal{F}_t\right] + \textit{FtDDVA}_{2-k}(t;T).$$

$\text{UCVA}_k(t;T)$ as a special case of $\text{FtDCVA}_k(t;T)$

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and

$$\mathbb{E}_{\mathbb{Q}}\big[\widehat{\Pi}_{2-k}^{(t,T]}|\mathcal{F}_t\big] = \mathbb{E}_{\mathbb{Q}}\big[\Pi_{2-k}^{(t,T]}|\mathcal{F}_t\big] + \text{FtDDVA}_{2-k}(t;T).$$

Hence, if party k were the investor, and if $\tau_k = +\infty$ the Unilateral CVA $0 \leq \text{UCVA}_k(t,T) := \text{FtDCVA}_k(t,T)$ would have to be paid by party $2-k$ to the default free party k at t to cover a potential default of party $2-k$ after t.

Structure of $\text{BFtDCVA}_k(t;T)$ I

Note the very important fact that always

$$
\begin{aligned}
\text{BFtDCVA}_k(t;T) &= \text{FtDCVA}_k(t;T) - \text{FtDDVA}_k(t;T) \\
&= \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A^-_{2-k}} \text{LGD}_{2-k} D(t,\tau_{2-k})(V_k(\tau_{2-k};T))^+ \middle| \mathcal{F}_t\right] \\
&- \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A^-_k} \text{LGD}_k D(t,\tau_k)(V_k(\tau_k;T))^- \middle| \mathcal{F}_t\right]
\end{aligned}
$$

Thus, for all $t \in [0,T]$, $\text{FtDCVA}_k(t;T)$ actually describes the t-risk neutral price of a derivative reveiling a complex structure, bought by party $2-k$ from party k. Hence, party $2-k$ pays $\text{FtDCVA}_k(t;T)$ to party k. The derivative is an European Call with strike zero, giving party $2-k$ the right (yet not the obligation) to "buy" party k's loss (or "residual value") $\mathbb{1}_{A^-_{2-k}} \cdot \text{LGD}_{2-k} \cdot V_k(\tau_{2-k};T)$ for price 0 at T.

Structure of BFtDCVA$_k(t;T)$ II

- By construction
  BFtDCVA$_k(t;T) \overset{(!)}{=} -$BFtDCVA$_{2-k}(t;T) =: -$BFtDDVA$_k(t;T)$ for all $0 \leq t < \min\{\tau_0, \tau_2\}$; reflecting an “agreed CCR price” between the two parties k and $2-k$. A symmetry of this type breaks down for the unilateral case since in general $0 \leq$ UCVA$_k(t;T) \neq -$UCVA$_{2-k}(t;T) \leq 0$. BFtDCVA$_k(\cdot;T)$ could change its sign very often (caused by an irregular oscillation, for example)!
- If the discounted MtM V_k decreases (in time), $(U_k)^- = (U_{2-k})^+$ increases. Consequently, if all other risk factors did not change, FtDDVA$_k$ would increase in time. Hence, given an increase of party k’s credit-spreads, the risk-neutral price of the defaultable claim to party k would increase as well (due to Brigo-Capponi)! So, would party k gain from an approximation to its own default?

Structure of BFtDCVA$_k(t;T)$ III

Consequently, to avoid this "DVA paradox" in first-to-default scenarios (by assuming the preclusion of the existence of arbitrage opportunities) one of the following assumptions is wrong and has to be reviewed:

- The validness of the Money Conservation Principle (MCP);
- The usefulness of the CCR clean ("risk-free") Full Two-Way Payment Rule under ISDA Master Agreements.

Here, the paper [1] suggests alternatives. In particular, we investigate the impact of substituting the CCR clean ("risk-free") Full Two-Way Payment Rule under ISDA Master Agreements through an extended "CCR including" version, suggested by ISDA in 2009, implying the need for a thorough understanding of accounting rules such as IFRS and US-GAAP though.

Structure of BFtDCVA$_k(t;T)$ IV

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Although we write "FtDCVA$_k(t;T)$" (or "FtDCVA$_t(k;T)$" in [1]), it always should be kept in mind that we actually are working with a very complex object, namely:

$$\boxed{\text{FtDCVA}_k(t, \text{LGD}_{2-k}, \tau_k, \tau_{2-k}, D(t,\tau_{2-k}), V_k(\tau_{2-k};T))}\,!$$

Remark (BFtDCVA as Difference of "Expected Losses")

BFtDCVA$_k(0;T) = EL_k - EL_{2-k}$, where

$$EL_k := \mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{A^-_{2-k}} LGD_{2-k} D(0,\tau_{2-k})(V_k(\tau_{2-k};T))^+\right].$$

FtDCVA$_k(0;T) = EL_k$ is also known as "Asset Charge" and FtDDVA$_k(0;T) = EL_{2-k}$ as "Liability Benefit".

Brainstorming

Recall the derivation of the (advanced) IRBA in Basel II . . .

Just the case $t = 0$

In the following, we consider the case $t = 0$ “only”. Why?

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The case $0 < t < T$ requires the use of a rather advanced mathematics including an in depth analysis of the conditional joint default process $\left(\mathbb{Q}\left(\tau_k \leq T \text{ and } \tau_{2-k} \leq \tau_k \middle| \mathcal{F}_t\right)\right)_{0 \leq t \leq T}$ under partial market information.

Just the case $t = 0$

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To cover dynamically changing stochastic dependence between all embedded risk factors, a truly dynamic copula model has to be constructed ($\rightsquigarrow$ Bielecki, Crépey, Jeanblanc et al).

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1. Counterparty Credit Risk (CCR)
2. On Netting Agreements
3. CCR Cash Flow Structure in a Financial Network
4. First-to-Default Credit Valuation Adjustments (FtDCVA)
5. UCVA in Basel III and the ISDA Formula
6. Risk Factors in FtDCVA: An Integrated View
7. Future Tasks

UCVA in Basel III - Part I

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Firstly we list a very restrictive case of a possible calculation of UCVA, encoded in the much too simple "CVA $=$ PD $*$ LGD $*$ EE" formula which however seems to be used often in financial institutes.

Proposition (Rough Approximation – Part I)

Assume that

(i) *party k will not default until T: $\tau_k := +\infty$;*

(ii) *LGD_{2-k} is constant and non-random;*

(iii) *$(U_k(0, \tau_{2-k}))^+ = D(0, \tau_{2-k}) \cdot (V_k(\tau_{2-k}; T))^+$ and τ_{2-k} are independent under $\mathbb{Q}$ (i. e., WWR or RWR is ignored completely).*

Then

$$UCVA_k(0; T) = \mathbb{Q}(\tau_{2-k} \leq T) \cdot LGD_{2-k} \cdot \mathbb{E}_{\mathbb{Q}}\left[(U_k(0, \tau_{2-k}))^+\right].$$

UCVA in Basel III - Part II

Suppose there exists a further random variable M (a “market risk factor”) so that $V_k(\tau_{2-k};T)$ is a function of M as well, $V_k(\tau_{2-k},M;T)$ say.

Proposition (Rough Approximation – Part II)

Assume that

(i) *Party k will not default until T: $\tau_k := +\infty$;*

(ii) *LGD_{2-k} is constant and non-random;*

(iii) *For all t $D(0,t)$ does not depend on M;*

(iv) *M and τ_{2-k} are independent under $\mathbb{Q}$.*

Then

$$UCVA_k(0;T) = LGD_{2-k} \int_0^T D(0,t)\mathbb{E}_{\mathbb{Q}}[(V_k(t,M;T))^+]\, dF^{\mathbb{Q}}_{\tau_{2-k}}(t),$$

where $F^{\mathbb{Q}}_{\tau_{2-k}}(t) := \mathbb{Q}(\tau_{2-k} \leq t)$ for all $t \in \mathbb{R}$ (unconditional df).

Proof.

Put $\Phi(t,m) := \mathbb{1}_{[0,T]}(t) \cdot \psi(t,m)$, where $(t,m)^\top \in \mathbb{R}^+ \times \mathbb{R}$ and $\psi(t,m) := D(0,t) \cdot (V_k(t,m;T))^+$. Let $F^{\mathbb{Q}}_{(\tau_{2-k},M)}$ denote the *bivariate* df of the random vector (τ_{2-k}, M) w.r.t. $\mathbb{Q}$. Then

$$\begin{aligned}
\text{UCVA}_k(0;T) &\overset{(i),(ii)}{=} \text{LGD}_{2-k}\mathbb{E}_{\mathbb{Q}}[\Phi(\tau_{2-k}, M)] \\
&= \text{LGD}_{2-k} \int_{\mathbb{R}^+\times\mathbb{R}} \Phi(t,m)\,dF^{\mathbb{Q}}_{(\tau_{2-k},M)}(t,m) \\
&\overset{(iv),\text{Fubini}}{=} \text{LGD}_{2-k} \int_{[0,T]} \left(\int_{\mathbb{R}} \psi(t,m)\,dF^{\mathbb{Q}}_{M}(m)\right) dF^{\mathbb{Q}}_{\tau_{2-k}}(t) \\
&\overset{(iii)}{=} \text{LGD}_{2-k} \int_0^T D(0,t)\mathbb{E}_{\mathbb{Q}}[(V_k(t,M;T))^+]\,dF^{\mathbb{Q}}_{\tau_{2-k}}(t).
\end{aligned}$$

□

Wrong-Way Risk and Right-Way Risk I

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$\text{EE}_k^{(M)}(t) := \mathbb{E}_{\mathbb{Q}}[(V_k(t, M; T))^+]$ is known as party k's Expected Exposure at t. In general it can be identified by MC simulation only.

Wrong-Way Risk and Right-Way Risk I

BaFin

$\mathrm{EE}_k^{(M)}(t) := \mathbb{E}_{\mathbb{Q}}[(V_k(t,M;T))^+]$ is known as party k's Expected Exposure at t. In general it can be identified by MC simulation only.

The situation where $\mathbb{Q}(\tau_{2-k} \leq t)$ is positively dependent on $\mathrm{EE}_k^{(M)}(t)$, is referred to as Wrong-Way Risk (WWR). In the case of WWR, there is a tendency for party $2-k$ to default when party k's exposure to party $2-k$ is relatively high. The situation where $\mathbb{Q}(\tau_{2-k} \leq t)$ is negatively dependent on $\mathrm{EE}_k^{(M)}(t)$ is referred to as Right-Way Risk (RWR). In the case of RWR, there is a tendency for party $2-k$ to default when party k's exposure to party $2-k$ is relatively low (cf. [5], [6]).

Wrong-Way Risk and Right-Way Risk II

BaFin

A simple way to include WWR is to use the “alpha” multiplier α of Basel II to increase $\text{EE}_k^{(M)}(t)$ if $\text{EE}_k^{(M)}(t)$ and $\mathbb{Q}(\tau_{2-k} \leq t)$ are assumed to be independent. The effect of α is to increase UCVA. Basel II sets $\alpha := 1.4$ or allows banks to use their own models, with $\alpha \geq 1.2$. This means that, at minimum, the UCVA has to be 20% higher than that one given in the case of the independence assumption. If a bank does not have its own model for WWR it has to be 40% higher. Estimates of α reported by banks range from 1.07 to 1.10 (cf. [6]).

Is the ISDA formula (Para 98) of Basel III true?

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Technical Remark

Regarding the calculation of $UCVA_k(0;T)$ in Basel III (para 98), observe that the integral in the above Proposition in fact is a Lebesgue-Stieltjes integral. Hence, if $t \mapsto EE_k^{(M)}(t)$ were not continuous (in time) and if it oscillated too strongly, that integral would not necessarily be a Riemann-Stieltjes integral, implying that we seemingly cannot simply approximate it numerically through a Riemann-Stieltjes sum of the type

$$
\begin{aligned}
UCVA_k(0;T) &\approx \sum_{i=1}^{n} D(0,t_i^*) \cdot EE_k^{(M)}(t_i^*) \cdot \left(F^{\mathbb{Q}}_{\tau_{2-k}}(t_i) - F^{\mathbb{Q}}_{\tau_{2-k}}(t_{i-1})\right) \\
&= \sum_{i=1}^{n} D(0,t_i^*) \cdot EE_k^{(M)}(t_i^*) \cdot \mathbb{Q}(t_{i-1} < \tau_{2-k} \le t_i), \quad (3)
\end{aligned}
$$

Basel III UCVA slightly modified

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where $0 = t_0 < \ldots < t_n = T$ and $t_i^* := \frac{t_{i-1}+t_i}{2}$. However:

Corollary

Assume that

(i) *The assumptions* (i), (ii) *and* (iv) *of the previous Proposition are satisfied;*

(ii) *For all* $i = 1, \ldots, n$, *for all* $t \in [t_{i-1}, t_i]$, $\mathbb{Q}(\tau_{2-k} > t) = \exp\big(-\lambda_{2-k}^{(i)}\, t\big)$, *where* $\lambda_{2-k}^{(i)} > 0$ *is a constant;*

(iii) *For all* $i = 1, \ldots, n$, *for all* $t \in [t_{i-1}, t_i]$, $r(t) \equiv r_i$ *is constant;*

(iv) LGD_{2-k} *is calibrated from a CDS curve with constant CDS spread* $s_{2-k}^{(i)}$ *on each* $[t_{i-1}, t_i]$.

Then $s_{2-k}^{(i)} = \lambda_{2-k}^{(i)} \cdot LGD_{2-k}$ *("Credit Triangle"), and*

$$UCVA_k(0;T) = \sum_{i=1}^{n} s_{2-k}^{(i)} \int_{t_{i-1}}^{t_i} e^{-r_i\, t} EE_k^{(M)}(t) \exp\left(-\frac{s_{2-k}^{(i)} t}{LGD_{2-k}}\right) dt.$$

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CVA risk in Basel III (Para 99)

Assuming the validity of approximation (3) of Basel III together with the “spread representation”

$$\mathbb{Q}(t^*_{i-1} < \tau_{2-k} \leq t^*_i) = e\big(s^{(i-1)}_{2-k}, t^*_{i-1}\big) - e\big(s^{(i)}_{2-k}, t^*_i\big),$$

where $e(s,t) := \exp(-s \cdot t/\mathrm{LGD}_{2-k})$, a Taylor series approximation of 2^{nd} order leads to the so called “CVA risk” of Basel III, i. e., to a delta/gamma approximation for $\mathrm{UCVA}_k(0;T)$, viewed as a function $f(s_{2-k})$ of the n-dimensional spread vector $s_{2-k} \equiv (s^{(1)}_{2-k}, \ldots s^{(n)}_{2-k})^\top$ only:

$$f(s_{2-k}+h) - f(s_{2-k}) \overset{(\|h\| \text{ small })}{\approx}$$

$$\sum_{i=1}^{n} D(0,t^*_i)\mathrm{EE}^{(M)}_k(t^*_i)\, h_i\big(t^*_i e\big(s^{(i)}_{2-k}, t^*_i\big) - t^*_{i-1} e\big(s^{(i-1)}_{2-k}, t^*_{i-1}\big)\big) \; +$$

$$\frac{1}{2\,\mathrm{LGD}_{2-k}} \sum_{i=1}^{n} D(0,t^*_i)\mathrm{EE}^{(M)}_k(t^*_i)\, h_i^2\big(t^{*2}_{i-1} e\big(s^{(i-1)}_{2-k}, t^*_{i-1}\big) - t^{*2}_i e\big(s^{(i)}_{2-k}, t^*_i\big)\big).$$

CVA risk in Basel III: key shortfalls I

A Fundamental Review of CVA risk and its capitalisation should particularly treat the following key shortfalls:

(i) CVA risk (and hedges) extend far beyond the risk of credit spread changes. It includes all risk factors that drive the underlying counterparty exposures as well as dependent interactions between counterparty exposures and the credit spreads of the counterparties (and their underyings). By solely focusing on credit spreads, the Basel III UCVA VaR and stressed VaR measures in its advanced approach for determining a CVA risk charge do not reflect the real risks that drive the P&L and earnings of institutes. Moreover, banks typically hedge these non-credit-spread risk factors. The Basel III capital calculation does not include these hedges.

CVA risk in Basel III: key shortfalls II

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(ii) The non-negligible and non-trivial problem of a more realistic inclusion of WWR should be analysed deeply. In particular, the "alpha" multiplier $1.2 \leq \alpha$ should be revisited, and any unrealistic independence assumption should be strongly avoided.

(iii) Credit and market risks in UCVA are not different from the same risks, embedded in many other trading positions such as corporate bonds, CDSs, or equity derivatives. CVA risk can be seen as just another source of market risk. Consequently, it should be managed within the trading book. Basel III requires that the CVA risk charge is calculated on a stand alone basis, separated from the trading book. This seems to be an artificial segregation. A suitable approach would be to include UCVA and all of its hedges into the trading book capital calculation.

CVA risk in Basel III: key shortfalls III

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(iv) Basel III requires the calculation of UCVA VaR and stressed VaR based on regulatory IMM expected exposure profiles and stressed exposure profiles. According to ISDA, this seems to be inconsistent with the way institutes mark to market, measure risk of and hedge their UCVA.

(v) Basel III has not considered **DVA risk** yet. This should be analysed more deeply.

(vi) Our investigation clearly shows that BFtDCVA should be implemented as opposed to UCVA as it is the case in the current version of Basel III.

(vii) Points not discussed here, yet heavy work in progress (in RMG): Para 75 of Basel III: how to derecognise own-credit related gains and losses from derivatives in Common Equity Tier 1 Capital? The role of DVA here?

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Thank you for your attention!

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Thank you for your attention!

Are there any questions, comments or remarks?