Towards a Quantification of Model Risk in Derivatives Pricing

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Ernst & Young - QAS Discussion Meeting

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What is a Model?

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Our short presentation mostly sheds a few rays of light on model risk emanating from the use of derivatives pricing models.

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- In the EU, model risk management is located as part of the governance component of the annual *Supervisory Review and Evaluation Process (SREP)*. The associated standards are the subject of the ECB's *Targeted Review of Internal Models (TRIM)*, launched on 17 February 2017.

Regulatory Guidance II

	Reference	Statements
General requirements	MaRisk (BaFin)	▪ Instruction to verify the **adequacy of methods** and **approaches** and to take the **limitations** and **restrictions** of these **methods** and **approaches** into account
General requirements	CRR / CRD	▪ Instruction to **measure model risks** arising from the use of **internal models** (OpRisk) – **no explicit method/ approach** mentioned
General requirements	SREP	▪ Instruction to **assess model risk** of **internal models** ▪ Direct **link** between **model deficiencies** and **capital requirements** (ICAAP) – guidance for holding **risk capital**
General requirements	EBA: Prudent valuation	▪ Explicit instruction to **measure model risk** via **valuation adjustments** resulting from the use of **different valuation models** and **model calibrations**
Specific	OCC / FED	▪ General guidance on **model risk management** through a **framework** covering model **development** and **deployment**, **use**, **validation**, **governance**, **policies** and **control**, and **documentation**

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  - It is expected that the number of risk parameter models will double.

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  - TRIM stipulates *topics and models*, selected on the basis of supervisory knowledge of substantial issues requiring adjustment and experience of regulators with particular portfolios and models.

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- A First Approach to Measure Model Risk
- A Sample of Model Risk

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Morini 2011: Model risk is the possibility that a financial institution suffers losses due to *mistakes in the development and application of valuation models.* (**Design Approach**)

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This example just addresses one of the factors contributing to model risk: choice of a model for the underlying. It could easily be extended to cover further factors by enlarging the set of models so that also various calibration approaches are included.

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The "buyer" B is the borrower of the notional N. Hence, if the interest rates increase monotonically and exceed r^* between the date that the FRA is traded (i.e., at $t = 0$) and the date t that the FRA comes into effect, bank B will be protected, else B would have to pay the difference between the fixed FRA rate r^* and the actual rate (LIBOR, say), as a percentage of the notional N.

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$$C(t) := D(t, 2t) \cdot tN \left(L(t, 2t) - r^*\right)$$

in cash (paid by bank S to bank B), where for any $0 \leq u \leq v$

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To replicate the cash flow of the FRA bank B enters into a further (uncollateralised) deal with a third market participant; CP C, say.

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	FRA long	***FRA replication***
At $t = 0$	Bank B buys an FRA on $N = 1$ from bank S. $\Rightarrow$ cash-flow $= 0$	Bank S borrows $D(0,t)$ until t from CP C (at $L(0,t)$) and lends $D(0,2t)\,(1+tr^*)$ to CP C until $2t$ (at $L(0,2t)$). $\Rightarrow$ cash-flow $= D(0,t) - D(0,2t)\,(1+tr^*)$
At t	Bank B borrows $N = 1$ from bank S (at $L(t,2t)$) and receives $C(t)$ from the FRA $\Rightarrow$ cash-flow $= 1 + C(t)$	Bank S pays 1 to CP C. Bank S borrows $N = 1$ from CP C again, until $2t$ (at $L(t,2t)$). $\Rightarrow$ cash-flow $= -1 + 1 = 0\,(!)$
At $2t$	Bank B pays $1 + tL(t,2t)$ to bank S. $\Rightarrow$ cash-flow $= -\,(1 + tL(t,2t))$	Bank S pays $1 + tL(t,2t)$ and receives $1 + tr^*$ from CP C. $\Rightarrow$ cash-flow $= t\,(r^* - L(t,2t))$

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$$\text{NPV}_{S,C} = \text{NPV}\left((0, 1, -(1 + tL(t,2t))\right) = D(0,t) - D(0,2t)(1 + tL(t,2t)) \,.$$

Note that both NPVs involve the random variable $L(t,2t)$. However, since $\text{NPV}_{S,C}$ does not require the calculation of $D(t,2t)$, the calculation of $\text{NPV}_{S,C}$ in fact arises from two *equalities* (and does not require “approximating” martingale methods as opposed to the case of $\text{NPV}_{B,S}$).

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Is all that true?

A Partial Answer - The Multi-curve Approach

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Since mid-2007 banks became increasingly reluctant to lend to each other because of counterparty default concerns. As a result, LIBOR quotes started to rise relative to other rates that involved very little default risk, implying that LIBOR started to incorporate a credit spread, reflecting the possibility that the borrowing bank may default.

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$$F^{\text{OIS}}(t,t,2t) \neq L(t,2t).$$

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Are there any questions, comments or remarks?