MATHEMATICS

Is Hahn-Banach equivalent to the ultrafilter lemma in ZF

Asked 12 days ago Modified 12 days ago Viewed 83 times

3

I know that the ultrafilter lemma is weaker than the axiom of choice (in ZF) And that in order to prove Choice in ZF from the ultrafilter lemma we need the Krein-Milman theorem so $UF + KM = AC$

Furthermore, I know the Hahn-Banach theorem is weaker than choice but in order to get choice from the Hahn-Banach theorem we need (again) Krein-Milman (in ZF). so $HB + KM = AC$.

(And I please complete ignorance of mathematical logic but...) Doesn't that imply that the Hahn-Banach theorem is logically equivalent to the ultrafilter lemma in ZF?

Or if not, does it then imply that we can use a statement that is weaker than Krein-Milman (call such a statement S) such that $UF + S = AC$?

So the question is either is $HB = UF$ or if not, what is S?

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edited Mar 2 at 14:13
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asked Mar 2 at 0:08
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To add to Asaf's answer the Hahn Banach theorem is equivalent to the fact that every finitely additive measures on subalgebras of boolean algebras can be extended to the whole algebra
– Math_Images_Only Mar 2 at 0:35

1 Answer

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4

No. Hahn–Banach is weaker than the Ultrafilter Lemma.

> *Pincus, David*, **Independence of the prime ideal theorem from the Hahn Banach theorem**, Bull. Am. Math. Soc. 78, 766-770 (1972). ZBL0257.02051.

In order to use Hahn–Banach to augment Krein–Milman to the full axiom of choice, you need a stronger version of the Krein–Milman theorem.

> *Bell, J. L.; Fremlin, D. H.*, **A geometric form of the axiom of choice**, Fundam. Math. 77, 167-170 (1973). ZBL0244.46014.

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answered Mar 2 at 0:23

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