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Arbitrage Free Credit Valuation Adjustments

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Agenda I

1. Common questions and Introduction
2. The mechanics of counterparty risk
   - General formula, Symmetry vs Asymmetry
   - Risk Free Closeout or Substitution Closeout?
   - Can we neglect first to default risk?
   - Contingent Credit Default Swap
3. Our modeling approach
4. 4 applications: Rates, Commodities Credit and Equity
   - Rates
   - Stressing underlying vols, credit spread vols, and correlations
   - Commodities
   - Credit
   - Equity
5. Counterparty Credit Risk and Collateral Margining
   - Collateralization and Gap Risk

Agenda II

- Margining Practice
- Close-Out Netting Rules
- Risk-Neutral Modelling of Bilateral CVA with Margining

6. Restructuring Counterparty Risk
   - Basel III and CVA
   - Margin Lending
   - Global Valuation

7. Conclusions and References
   - Conclusions
   - References

Some common questions 1

Q What is counterparty risk in general?

A *The risk taken on by an entity entering an OTC contract with a counterparty having a relevant default probability. As such, the counterparty might not respect its payment obligations.*

> *The counterparty credit risk is defined as the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows. An economic loss would occur if the transactions or portfolio of transactions with the counterparty has a positive economic value at the time of default.*

[Basel II, Annex IV, 2/A]

Some common questions

Q What is the difference between Credit VaR and CVA?

A *Credit VaR is a Value at Risk type measure, it measures a potential loss due to counterparty default. CVA stands for Credit Valuation Adjustment and is a price adjustment. CVA is obtained by pricing the counterparty risk component of a deal, similarly to how one would price a credit derivative.*

Q Are the methodologies for Credit VaR and CVA similar?

A *There are analogies but CVA needs to be more precise in general. Also, Credit VaR should use statistics under the physical measure whereas CVA should use statistics under the pricing measure*

Q What are the regulatory bodies involved?

A *There are many, for Credit VaR type measures it is mostly Basel II and now III, whereas for CVA we have IAS and ISDA.*

Q What is the focus of this presentation?

A *We will focus on CVA.*

Some common questions

Q When is valuation of counterparty risk CVA symmetric?

A *When we include the possibility that also the entity computing the counterparty risk adjustment may default, besides the counterparty itself.*

Q When is valuation of counterparty risk CVA asymmetric?

A *When the entity computing the counterparty risk adjustment considers itself default-free, and only the counterparty may default.*

Q Which one is computed usually for valuation adjustments?

A *Pre-crisis it used to be the asymmetric one; At the moment there is quite a debate*

Some common questions

Basel II on bilateral counterparty risk:

> *Unlike a firm's exposure to credit risk through a loan, where the exposure to credit risk is unilateral and only the lending bank faces the risk of loss, the counterparty credit risk creates a bilateral risk of loss: the market value of the transaction can be positive or negative to either counterparty to the transaction.* [Basel II, Annex IV, 2/A]

Some common questions

Q What impacts counterparty risk?

A *The OTC contract's underlying volatility, the correlation between the underlying and default of the counterparty, and the counterparty credit spreads volatility.*

Q Is it model dependent?

A *It is.*

Q What about *wrong way risk*?

A *The amplified risk when the reference underlying and the counterparty are strongly correlated in the wrong direction.*

Some common questions

Q What is collateral?

A *It is a guarantee (liquid and secure asset, cash) that is deposited in a collateral account in favour of the investor party facing the exposure. If the depositing counterparty defaults, thus not being able to fulfill payments associated to the above mentioned exposure, Collateral can be used by the investor to offset its loss.*

Q What is netting?

A *This is the agreement to net all positions towards a counterparty in the event of the counterparty default. This way positions with negative PV can be offset by positions with positive PV and counterparty risk is reduced. This has to do with the option on a sum being smaller than the sum of the options*

Q Is Counterparty risk CVA model dependent? **A** *It is.*

Some common questions

Q What is happening with Basel III?

A *Basel noticed that during the crisis only one third of losses due to counterparty risk were due to actual defaults. The remaining losses have been due to CVA mark to market losses. Hence the pricing of counterparty risk has been twice as dangerous as the risk itself.*

Q Then we should ”risk-measure” CVA itself?

A *Indeed there is a lot of discussion around Value at Risk of CVA. This is not traditional credit VaR of course. It is something much more sophisticated. It is a percentile on future possible losses due to future averse movements of the PRICING of counterparty risk*

For an introductory dialogue on Counterparty Risk see

CVA FAQ

D. Brigo (2011). Counterparty Risk FAQ: Credit VaR, CVA, DVA, Closeout, Netting, Collateral, Re-hypothecation, Wrong Way Risk, Basel, Funding, and Margin Lending. Available at damianobrigo.it, arXiv.org, ssrn.com, defaultrisk.com

General Notation

- We will call "investor" the party interested in the counterparty adjustment. This is denoted by "0"
- We will call "counterparty" the party with whom the investor is trading, and whose default may affect negatively the investor. This is denoted by "2" or "C".
- "1" will be used for the underlying name/risk factor(s) of the contract
- The counterparty's default time is denoted with τ_C and the recovery rate for unsecured claims with Rec_C (we often use $\mathrm{Lgd}_C := 1 - \mathrm{Rec}_C$).
- $\Pi_0(t, T)$ is the discounted payout without default risk seen by '0'(sum of all future cash flows between t and T, discounted back at t). $\Pi_2(t, T) = -\Pi_0(t, T)$ is the same quantity but seen from the point of view of '2'. When we omit the index 0 or 2 we mean '0'.

General Notation

- We define $NPV_0(t,T) = \mathbb{E}_t[\Pi(t,T)]$. When T is clear from the context we omit it and write NPV(t).
- $$\Pi(s,t) + D(s,t)\Pi(t,u) = \Pi(s,u)$$
- $$\mathbb{E}_0[D(0,u)NPV(u,T)] = \mathbb{E}_0[D(0,u)\mathbb{E}_u[\Pi(u,T)]] =$$
$$= \mathbb{E}_0[D(0,u)\Pi(u,T)] = NPV(0,T) - \mathbb{E}_0[\Pi(0,u)]$$
$$= NPV(0,T) - NPV(0,u)$$

Asymmetric/Unilateral counterparty risk

We now look into unilateral or asymmetric counterparty risk.

This is a situation where it is assumed that only the counterparty can default, whereas the investor doing the calculation is assumed to be default free.

Hence we will only consider here the default time $\tau_2 = \tau_C$ of the counterparty '2'. We will address the bilateral case later on.

The mechanics of Evaluating asymmetric counterparty risk

payoff under counterparty default risk

counterparty defaults after final maturity

original payoff of the instrument

counterparty defaults before final maturity

all cash flows before default
$\oplus$ recovery of the residual NPV at default if positive
$\ominus$ Total residual NPV at default if negative

General Formulation under Asymmetry

$$\Pi_0^D(t,T) = \mathbf{1}_{\tau_2>T}\Pi_0(t,T)$$
$$+\mathbf{1}_{t<\tau_2\leq T}\left[\Pi_0(t,\tau_2) + D(t,\tau_2)\left(REC_2\left(\mathrm{NPV}_0(\tau_2)\right)^+ - \left(-\mathrm{NPV}_0(\tau_2)\right)^+\right)\right]$$

This last expression is the general payoff seen from the point of view of '0' (Π_0, NPV_0) under unilateral counterparty default risk. Indeed,

1. if there is no early default, this expression reduces to first term on the right hand side, which is the payoff of a default-free claim.
2. In case of early default of the counterparty, the payments due before default occurs are received (second term)
3. and then if the residual net present value is positive only the recovery value of the counterparty REC_2 is received (third term),
4. whereas if it is negative it is paid in full by the investor (fourth term).

General Formulation under Asymmetry

If one simplifies the cash flows and takes the risk neutral expectation, one obtains the fundamental formula for the valuation of counterparty risk when the investor 0 is default free:

$$\mathbb{E}_t\left\{\Pi_0^D(t,T)\right\} = \mathbf{1}_{\{\tau_2>t\}}\mathbb{E}_t\left\{\Pi_0(t,T)\right\} - \mathbb{E}_{\mathrm{t}}\left\{\mathrm{LGD}_2\mathbf{1}_{\{\mathrm{t}<\tau_2\leq\mathrm{T}\}}\mathrm{D}(\mathrm{t},\tau_2)\left[\mathrm{NPV}_0(\tau_2)\right]^+\right\} \quad (*)$$

- First term : Value without counterparty risk.
- Second term : Unilateral Counterparty Valuation Adjustment
- $\mathrm{NPV}(\tau_C) = \mathbb{E}_{\tau_C}\left[\Pi(\tau_C,T)\right]$ is the value of the transaction on the counterparty default date. LGD = 1 - REC_counterparty.

$$\mathrm{UCVA}_0 = \mathbb{E}_{\mathrm{t}}\left\{\mathrm{LGD}_2\mathbf{1}_{\{\mathrm{t}<\tau_2\leq\mathrm{T}\}}\mathrm{D}(\mathrm{t},\tau_2)\left[\mathrm{NPV}_0(\tau_2)\right]^+\right\}$$

Proof of the formula

In the proof we omit indices: $\tau = \tau_2$, REC=REC$_2$, LGD=LGD$_2$, NPV=NPV$_0$, $\Pi = \Pi_0$. The proof is obtained easily putting together the following steps. Since

$$1_{\{\tau>t\}}\Pi(t,T) = 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t<\tau\leq T\}}\Pi(t,T)$$

we can rewrite the terms inside the expectation in the right hand side of the simplified formula (*) as

$$\begin{aligned}
& \mathbf{1}_{\{\tau>t\}}\Pi(t,T) - \left\{LGD\mathbf{1}_{\{t<\tau\leq T\}}D(t,\tau)\left[\mathrm{NPV}(\tau)\right]^+\right\} \\
& = 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t<\tau\leq T\}}\Pi(t,T) \\
+ \quad & \{(\mathrm{Rec}-1)[1_{\{t<\tau\leq T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+]\} \\
= \quad & 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t<\tau\leq T\}}\Pi(t,T) \\
+ \quad & \mathrm{Rec}\ \ 1_{\{t<\tau\leq T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+ - 1_{\{t<\tau\leq T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+
\end{aligned}$$

Conditional on the information at τ the second and the fourth terms are equal to

Proof (cont'd)

$$
\begin{aligned}
& E_\tau[1_{\{t<\tau\leq T\}}\Pi(t,T) - 1_{\{t<\tau\leq T\}}D(t,\tau)(\mathrm{NPV}(\tau))^+] \\
= \; & E_\tau[1_{\{t<\tau\leq T\}}[\Pi(t,\tau) + D(t,\tau)\Pi(\tau,T) - D(t,\tau)(E_\tau[\Pi(\tau,T)])^+]] \\
= \; & 1_{\{t<\tau\leq T\}}[\Pi(t,\tau) + D(t,\tau)E_\tau[\Pi(\tau,T)] - D(t,\tau)(E_\tau[\Pi(\tau,T)])^+] \\
= \; & 1_{\{t<\tau\leq T\}}[\Pi(t,\tau) - D(t,\tau)(E_\tau[\Pi(\tau,T)])^-] \\
= \; & 1_{\{t<\tau\leq T\}}[\Pi(t,\tau) - D(t,\tau)(E_\tau[-\Pi(\tau,T)])^+] \\
= \; & 1_{\{t<\tau\leq T\}}[\Pi(t,\tau) - D(t,\tau)(-\mathrm{NPV}(\tau))^+]
\end{aligned}
$$

since

$$1_{\{t<\tau\leq T\}}\Pi(t,T) = 1_{\{t<\tau\leq T\}}\{\Pi(t,\tau) + D(t,\tau)\Pi(\tau,T)\}$$

and $f = f^+ - f^- = f^+ - (-f)^+$.

Proof (cont'd)

Then we can see that after conditioning the whole expression of the original long payoff on the information at time τ and substituting the second and the fourth terms just derived above, the expected value with respect to $\mathcal{F}_t$ coincides exactly with the one in our simplified formula (*) by the properties of iterated expectations by which $\mathbb{E}_t[X] = \mathbb{E}_t[\mathbb{E}_\tau[X]]$.

What we can observe

- Including counterparty risk in the valuation of an otherwise default-free derivative $\Longrightarrow$ credit (hybrid) derivative.

- The inclusion of counterparty risk adds a level of optionality to the payoff.
  In particular, model independent products become model dependent also in the underlying market.
  $\Longrightarrow$ **Counterparty Risk analysis incorporates an opinion about the underlying market dynamics and volatility.**

The point of view of the counterparty "2"

The deal from the point of view of '2', while staying in a world where only '2" may default.

$$\Pi_2^D(t,T) = \mathbf{1}_{\tau_2 > T}\Pi_2(t,T)$$
$$+\mathbf{1}_{t<\tau_2\leq T}\left[\Pi_2(t,\tau_2) + D(t,\tau_2)\left((\mathrm{NPV}_2(\tau_2))^+ - REC_2\left(-\mathrm{NPV}_2(\tau_2)\right)^+\right)\right]$$

This last expression is the general payoff seen from the point of view of '2' (Π_2, NPV_2) under unilateral counterparty default risk. Indeed,

1. if there is no early default, this expression reduces to first term on the right hand side, which is the payoff of a default-free claim.
2. In case of early default of the counterparty '2", the payments due before default occurs go through (second term)
3. and then if the residual net present value is positive to the defaulted '2', it is received in full from '0' (third term),
4. whereas if it is negative, only the recovery fraction REC_2 it is paid to '0' (fourth term).

The point of view of the counterparty "2"

The above formula simplifies to

$$\mathbb{E}_t\left\{\Pi_2^D(t,T)\right\} = \mathbf{1}_{\tau_2>t}\mathbb{E}_t\left\{\Pi_2(t,T)\right\} + \mathbb{E}_t\left\{\mathrm{LGD}_2\mathbf{1}_{t<\tau_2\leq T}\mathrm{D}(t,\tau_2)\left[-\mathrm{NPV}_2(\tau_2)\right]^+\right\}$$

and the adjustment term with respect to the risk free price $\mathbb{E}_t\left\{\Pi_2(t,T)\right\}$ is called

UNILATERAL DEBIT VALUATION ADJUSTMENT

$$\mathrm{UDVA}_2(t) = \mathbb{E}_t\left\{\mathrm{LGD}_2\mathbf{1}_{\{t<\tau_2\leq T\}}\mathrm{D}(t,\tau_2)\left[-\mathrm{NPV}_2(\tau_2)\right]^+\right\}$$

We note that $\mathrm{UDVA}_2 = \mathrm{UCVA}_0$.
Notice also that in this universe $\mathrm{UDVA}_0 = \mathrm{UCVA}_2 = 0$.

Including the investor default or not?

Often the investor, when computing a counterparty risk adjustment, considers itself to be default-free. This can be either a unrealistic assumption or an approximation for the case when the counterparty has a much higher default probability than the investor.

If this assumption is made when no party is actually default-free, the valuation adjustment is asymmetric: if “2” were to consider itself as default free and “0” as counterparty, and if “2” computed the counterparty risk adjustment, this would not be the opposite of the one computed by “0” in the straight case.

Also, the total NPV including counterparty risk is similarly asymmetric, in that the total value of the position to “0” is not the opposite of the total value of the position to “2”.

Including the investor default or not?

We get back symmetry if we allow for default of the investor in computing counterparty risk. This also results in an adjustment that is cheaper to the counterparty “2”.

The counterparty “2” may then be willing to ask the investor “0” to include the investor default event into the model, when the Counterparty risk adjustment is computed by the investor

The case of symmetric counterparty risk

Suppose now that we allow for both parties to default. Counterparty risk adjustment allowing for default of "0"?
"0" : the investor; "2": the counterparty;
("1": the underlying name/risk factor of the contract).
τ_0, τ_2: default times of "0" and "2". T: final maturity
We consider the following events, forming a partition

Four events ordering the default times

$$
\begin{aligned}
A &= \{\tau_0 \leq \tau_2 \leq T\} \qquad E = \{T \leq \tau_0 \leq \tau_2\} \\
B &= \{\tau_0 \leq T \leq \tau_2\} \qquad F = \{T \leq \tau_2 \leq \tau_0\} \\
C &= \{\tau_2 \leq \tau_0 \leq T\} \\
D &= \{\tau_2 \leq T \leq \tau_0\}
\end{aligned}
$$

The case of symmetric counterparty risk

$$\Pi_0^D(t,T) = \mathbf{1}_{E\cup F}\Pi_0(t,T)$$
$$+\mathbf{1}_{C\cup D}\left[\Pi_0(t,\tau_2) + D(t,\tau_2)\left(REC_2\left(\mathrm{NPV}_0(\tau_2)\right)^+ - \left(-\mathrm{NPV}_0(\tau_2)\right)^+\right)\right]$$
$$+\mathbf{1}_{A\cup B}\left[\Pi_0(t,\tau_0) + D(t,\tau_0)\left(\left(\mathrm{NPV}_0(\tau_0)\right)^+ - REC_0\left(-\mathrm{NPV}_0(\tau_0)\right)^+\right)\right]$$

1. If no early default $\Rightarrow$ payoff of a default-free claim (1st term).
2. In case of early default of the counterparty, the payments due before default occurs are received (second term),
3. and then if the residual net present value is positive only the recovery value of the counterparty REC_2 is received (third term),
4. whereas if negative, it is paid in full by the investor (4th term).
5. In case of early default of the investor, the payments due before default occurs are received (fifth term),
6. and then if the residual net present value is positive it is paid in full by the counterparty to the investor (sixth term),
7. whereas if it is negative only the recovery value of the investor REC_0 is paid to the counterparty (seventh term).

The case of symmetric counterparty risk

$$\mathbb{E}_t\left\{\Pi_0^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi_0(t,T)\right\} + \mathrm{DVA}_0(t) - \mathrm{CVA}_0(t)$$

$$\mathrm{DVA}_0(t) = \mathbb{E}_{\mathrm{t}}\left\{\mathrm{LGD}_0 \cdot \mathbf{1}(\mathrm{t} < \tau^{1\mathrm{st}} = \tau_0 < \mathrm{T}) \cdot \mathrm{D}(\mathrm{t},\tau_0) \cdot [-\mathrm{NPV}_0(\tau_0)]^+\right\}$$

$$\mathrm{CVA}_0(t) = \mathbb{E}_{\mathrm{t}}\left\{\mathrm{LGD}_2 \cdot \mathbf{1}(\mathrm{t} < \tau^{1\mathrm{st}} = \tau_2 < \mathrm{T}) \cdot \mathrm{D}(\mathrm{t},\tau_2) \cdot [\mathrm{NPV}_0(\tau_2)]^+\right\}$$

$$1(A \cup B) = 1(t < \tau^{1st} = \tau_0 < T), \quad 1(C \cup D) = 1(t < \tau^{1st} = \tau_2 < T)$$

- Obtained simplifying the previous formula and taking expectation.
- 2nd term : adj due to scenarios $\tau_0 < \tau_2$. This is positive to the investor 0 and is called ”Debit Valuation Adjustment” (DVA)
- 3d term : Counterparty risk adj due to scenarios $\tau_2 < \tau_0$
- Bilateral Valuation Adjustment as seen from 0: $\mathrm{BVA}_0 = \mathrm{DVA}_0 - \mathrm{CVA}_0$.
- If computed from the opposite point of view of “2” having counterparty “0”, $\mathrm{BVA}_2 = -\mathrm{BVA}_0$. Symmetry.

The case of symmetric counterparty risk

Strange consequences of the formula new mid term, i.e. DVA

- credit quality of investor WORSENS $\Rightarrow$ books POSITIVE MTM
- credit quality of investor IMPROVES $\Rightarrow$ books NEGATIVE MTM
- Citigroup in its press release on the first quarter revenues of 2009 reported a *positive* mark to market due to its *worsened* credit quality: "Revenues also included [...] a net 2.5$ billion positive CVA on derivative positions, excluding monolines, mainly due to the widening of Citi's CDS spreads"

Inconsistency of Accounting and Capital Requirements regulation

NO DVA: Basel III, page 37, July 2011 release

”This CVA loss is calculated without taking into account any offsetting debit valuation adjustments which have been deducted from capital under paragraph 75.”

YES DVA: FAS 157

”Because nonperformance risk (the risk that the obligation will not be fulfilled) includes the reporting entity’s credit risk, the reporting entity should consider the effect of its credit risk (credit standing) on the fair value of the liability in all periods in which the liability is measured at fair value under other accounting pronouncements FAS 157 (see also IAS 39)

Inconsistency of Accounting and Capital Requirements regulation

OH WELL!! Stefan Walter says:

The potential for perverse incentives resulting from profit being linked to decreasing creditworthiness means capital requirements cannot recognise it, says Stefan Walter, secretary-general of the Basel Committee: The main reason for not recognising DVA as an offset is that it would be inconsistent with the overarching supervisory prudence principle under which we do not give credit for increases in regulatory capital arising from a deterioration in the firms own credit quality.

What are some of the Top Banks doing?

The case of symmetric counterparty risk: DVA?

More recently, From the Wall Street Journal

`October 18, 2011, 3:59 PM ET. Goldman Sachs Hedges Its Way to Less Volatile Earnings.`

Goldmans DVA gains in the third quarter totaled $450 million, about $300 million of which was recorded under its fixed-income, currency and commodities trading segment and another $150 million recorded under equities trading.

That amount is comparatively smaller than the $1.9 billion in DVA gains that J.P. Morgan Chase and Citigroup each recorded for the third quarter. Bank of America reported $1.7 billion of DVA gains in its investment bank.

Analysts estimated that Morgan Stanley will record $1.5 billion of net DVA gains when it reports earnings on Wednesday [...]

The case of symmetric counterparty risk: DVA?

How can DVA be hedged? One should sell protection on oneself, an impossible feat, unless one buys back bonds that he had issued earlier. This may be hard to implement, though.

Most times DVA is hedged by proxying. Instead of selling protection on oneself, one sells protection on a number of names that one thinks are highly correlated to oneself.

The case of symmetric counterparty risk: DVA?

Again from the WSJ article above:

[...] Goldman Sachs CFO David Viniar said Tuesday that the company attempts to hedge [DVA] using a basket of different financials.
A Goldman spokesman confirmed that the company did this by selling CDS on a range of financial firms. [...] Goldman wouldnt say what specific financials were in the basket, but Viniar confirmed [...] that the basket contained a peer group. Most would consider peers to Goldman to be other large banks with big investment-banking divisions, including Morgan Stanley, J.P. Morgan Chase, Bank of America, Citigroup and others. The performance of these companies bonds would be highly correlated to Goldmans.

This can approximately hedge the spread risk of DVA, but not the jump to default risk. Merrill hedging DVA risk by selling protection on Lehman would not have been a good idea.

The case of symmetric counterparty risk: DVA?

When allowing for the investor to default: symmetry

- DVA: One more term with respect to the asymmetric case.
- depending on credit spreads and correlations, the total adjustment to be subtracted (CVA-DVA) can now be either positive or negative. In the asymmetric case it can only be positive.
- Ignoring the symmetry is clearly more expensive for the counterparty and cheaper for the investor.
- Hedging DVA is difficult. Hedging “by peers” ignores jump to default risk
- We assume the asymmetric case in most of the numerical presentations
- WE TAKE THE POINT OF VIEW OF ‘0” from now on, so we omit the subscript ‘0’. We denote the counterparty as ‘2” or ‘C”.

Closeout: Substitution (ISDA?) VS Risk Free

When we computed the bilateral adjustment formula from

$$\begin{aligned}
&\Pi_0^D(t,T) = \mathbf{1}_{E\cup F}\Pi_0(t,T) \\
&+\mathbf{1}_{C\cup D}\left[\Pi_0(t,\tau_2) + D(t,\tau_2)\left(REC_2\left(\mathrm{NPV}_0(\tau_2)\right)^+ - \left(-\mathrm{NPV}_0(\tau_2)\right)^+\right)\right] \\
&+\mathbf{1}_{A\cup B}\left[\Pi_0(t,\tau_0) + D(t,\tau_0)\left(\left(-\mathrm{NPV}_2(\tau_0)\right)^+ - REC_0\left(\mathrm{NPV}_2(\tau_0)\right)^+\right)\right]
\end{aligned}$$

(where we now substituted $\mathrm{NPV}_0 = -\mathrm{NPV}_2$ in the last two terms) we used the risk free NPV upon the first default, to close the deal. But what if upon default of the first entity, the deal needs to be valued by taking into account the credit quality of the surviving party? What if we make the substitutions

$$\mathrm{NPV}_0(\tau_2) \to \mathrm{NPV}_0(\tau_2) + \mathrm{UDVA}_0(\tau_2)$$

$$\mathrm{NPV}_2(\tau_0) \to \mathrm{NPV}_2(\tau_0) + \mathrm{UDVA}_2(\tau_0)?$$

Closeout: Substitution (ISDA?) VS Risk Free

This seems to be supported by ISDA.

ISDA (2009) Close-out Amount Protocol.

”In determining a Close-out Amount, the Determining Party may consider any relevant information, including, without limitation, one or more of the following types of information: (i) quotations (either firm or indicative) for replacement transactions supplied by one or more third parties that may take into account the creditworthiness of the Determining Party at the time the quotation is provided”

This makes valuation more continuous: upon default we still price including the DVA, as we were doing before default.

Closeout: Substitution (ISDA?) VS Risk Free

The final formula with subsitution closeout is quite complicated:

$$
\begin{aligned}
&\Pi_0^D(t,T) = \mathbf{1}_{E\cup F}\Pi_0(t,T) \\
&+\mathbf{1}_{C\cup D}\Big[\Pi_0(t,\tau_2) + D(t,\tau_2) \\
&\cdot\left(REC_2\left(\mathrm{NPV}_0(\tau_2) + \mathrm{UDVA}_0(\tau_2)\right)^+ - \left(-\mathrm{NPV}_0(\tau_2) - \mathrm{UDVA}_0(\tau_2)\right)^+\right)\Big] \\
&+\mathbf{1}_{A\cup B}\Big[\Pi_0(t,\tau_0) + D(t,\tau_0) \\
&\cdot\left(\left(-\mathrm{NPV}_2(\tau_0) - \mathrm{UDVA}_2(\tau_0)\right)^+ - REC_0\left(\mathrm{NPV}_2(\tau_0) + \mathrm{UDVA}_2(\tau_0)\right)^+\right)\Big]
\end{aligned}
$$

Closeout: Substitution (ISDA?) VS Risk Free

B. and Morini (2010)

We analyze the Risk Free closeout formula in Comparison with the Substitution Closeout formula using a Zero coupon bond as a contract and in two cases:

1. Default of ‘0’ and ‘2” are independent
2. Default of ‘0’ and ‘2” are co-monotonic

Suppose ‘0’ (the lender) holds the bond,
and ‘2’ (the borrower) will pay the notional 1 at maturity T.

The risk free price of the bond at time 0 to ’0’ is denoted by $P(0, T)$.

Closeout: Substitution (ISDA?) VS Risk Free

If we assume deterministic interest rates, the above formulas reduce to

$$P^{D,Subs}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + REC_2\mathbb{Q}(\tau_2 \leq T)]$$

$$P^{D,Free}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + \mathbb{Q}(\tau_0 < \tau_2 < T) + REC_2\mathbb{Q}(\tau_2 \leq \min(\tau_0, T))]$$

$$= P(0,T)[\mathbb{Q}(\tau_2 > T) + REC_2\mathbb{Q}(\tau_2 \leq T) + LGD_2\mathbb{Q}(\tau_0 < \tau_2 < T)]$$

Credit Risk of the Lender

We see an important drawback of the risk free closeout in this case: The adjusted price of the bond DEPENDS ON THE CREDIT RISK OF THE LENDER '0' IF WE USE THE RISK FREE CLOSEOUT. This is counterintuitive and undesirable. From this point of view the Substitution Closeout is superior.

Closeout: Substitution (ISDA?) VS Risk Free

Co-Monotonic Case

If we assume the default of '0' and '2' to be co-monotonic, and the spread of the lender '0" to be larger, we have that the lender '0" defaults first in ALL SCENARIOS (e.g. '2' is a subsidiary of '0', or a company whose well being is completely driven by '0': '2' is a trye factory whose only client is car producer '0"). In this case

$$P^{D,Subs}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + REC_2\mathbb{Q}(\tau_2 \leq T)]$$

$$P^{D,Free}(0,T) = P(0,T)[\mathbb{Q}(\tau_2 > T) + \mathbb{Q}(\tau_2 < T)] = P(0,T)$$

Risk free closeout is correct. Either '0" does not default, and then '2" does not default either, or if '0" defaults, at that precise time 2 is solvent, and 0 recovers the whole payment. Credit risk of '2" should not impact the deal.

Closeout: Substitution (ISDA?) VS Risk Free

Contagion. What happens at default of the Lender

$$P^{D,Subs}(t,T) = P(t,T)[\mathbb{Q}_t(\tau_2 > T) + REC_2\mathbb{Q}_t(\tau_2 \leq T)]$$

$$P^{D,Free}(t,T) = P^{D,Subs}(t,T) + P(t,T)LGD_2\mathbb{Q}_t(\tau_0 < \tau_2 < T)$$

We focus on two cases:

- τ_0 and τ_2 are independent. Take $t < T$.

$$\mathbb{Q}_{t-\Delta t}(\tau_0 < \tau_2 < T) \mapsto \quad \{\tau_0 = t\} \quad \mapsto \mathbb{Q}_{t+\Delta t}(\tau_2 < T)$$

  and this effect can be quite sizeable.
- τ_0 and τ_2 are comonotonic. Take an example where $\tau_0 = t < T$ implies $\tau_2 = u < T$ with $u > t$. Then

$$\mathbb{Q}_{t-\Delta t}(\tau_2 > T) \mapsto \quad \{\tau_0 = t, \tau_2 = u\} \quad \mapsto 0$$

$$\mathbb{Q}_{t-\Delta t}(\tau_2 \leq T) \mapsto \quad \{\tau_0 = t, \tau_2 = u\} \quad \mapsto 1$$

$$\mathbb{Q}_{t-\Delta t}(\tau_0 < \tau_2 < T) \mapsto \quad \{\tau_0 = t, \tau_2 = u\} \quad \mapsto 1$$

Closeout: Substitution (ISDA?) VS Risk Free

Let us put the pieces together:

- τ_0 and τ_2 are independent. Take $t < T$.

$$P^{D,Subs}(t-\Delta t, T) \mapsto \quad \{\tau_0 = t\} \mapsto \quad \text{no change}$$

$$P^{D,Free}(t-\Delta t, T) \mapsto \quad \{\tau_0 = t\} \mapsto \quad \text{add } \mathbb{Q}_{t-\Delta t}(\tau_0 > \tau_2, \tau_2 < T)$$

  and this effect can be quite sizeable.

- τ_0 and τ_2 are comonotonic. Take an example where $\tau_0 = t < T$ implies $\tau_2 = u < T$ with $u > t$. Then

$$P^{D,Subs}(t-\Delta t, T) \mapsto \quad \{\tau_0 = t\} \mapsto \quad \text{subtract } X$$

$$X = LGD_2 P(t,T)\mathbb{Q}_{t-\Delta t}(\tau_2 > T)$$

$$P^{D,Free}(t-\Delta t, T) \mapsto \quad \{\tau_0 = t\} \mapsto \quad \text{no change}$$

Closeout: Substitution (ISDA?) VS Risk Free

The independence case: Contagion with Risk Free closeout

The Risk Free closeout shows that *upon default of the lender*, the mark to market to the lender itself jumps up, or equivalently **the mark to market to the borrower jumps down**. The effect can be quite dramatic.
The substitution closeout instead shows no such contagion, as the mark to market does not change upon default of the lender.

The co-monotonic case: Contagion with Substitution closeout

The Risk Free closeout behaves nicely in the co-monotonic case, and there is no change upon default of the lender.
Instead the Substitution closeout shows that *upon default of the lender* the mark to market to the lender jumps down, or equivalently **the mark to market to the borrower jumps up**.

Closeout: Substitution (ISDA?) VS Risk Free

Impact of an early default of the Lender

Dependence→ Closeout↓	independence	co-monotonicity
Risk Free	Negatively affects Borrower	No contagion
Substitution	No contagion	Further Negatively affects Lender

For a numerical case study see Brigo and Morini (2010, 2011).

A simplified formula without τ^{1st} for bilateral VA

Instead of the full bilateral formula, the industry at times uses the difference of two unilateral formulas. Replace (this is the risk free closeout case) the correct formula with first to default risk

$$\mathbb{E}_t\left\{\Pi_0^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi_0(t,T)\right\} + \mathrm{DVA}_0(t) - \mathrm{CVA}_0(t)$$

$$\mathrm{DVA}_0(t) = \mathbb{E}_{\mathrm{t}}\left\{\mathrm{LGD}_0 \cdot \mathbf{1}(\mathrm{t} < \tau^{\mathrm{1st}} = \tau_0 < \mathrm{T}) \cdot \mathrm{D}(\mathrm{t},\tau_0) \cdot [-\mathrm{NPV}_0(\tau_0)]^+\right\}$$

$$\mathrm{CVA}_0(t) = \mathbb{E}_{\mathrm{t}}\left\{\mathrm{LGD}_2 \cdot \mathbf{1}(\mathrm{t} < \tau^{\mathrm{1st}} = \tau_2 < \mathrm{T}) \cdot \mathrm{D}(\mathrm{t},\tau_2) \cdot [\mathrm{NPV}_0(\tau_2)]^+\right\}$$

with the approximated formula without first to default risk

$$\mathbb{E}_t\left\{\Pi_0^D(t,T)\right\} = \mathbb{E}_t\left\{\Pi_0(t,T)\right\} + \mathrm{UDVA}_0(t) - \mathrm{UCVA}_0(t)$$

$$= \mathbb{E}_t\left\{\Pi_0(t,T)\right\} + \mathrm{UCVA}_2(t) - \mathrm{UCVA}_0(t)$$

$$\mathrm{UDVA}_0(t) = \mathbb{E}_{\mathrm{t}}\left\{\mathrm{LGD}_0 \cdot \mathbf{1}(\mathrm{t} < \tau_0 < \mathrm{T}) \cdot \mathrm{D}(\mathrm{t},\tau_0) \cdot [-\mathrm{NPV}_0(\tau_0)]^+\right\}$$

$$\mathrm{UCVA}_0(t) = \mathbb{E}_{\mathrm{t}}\left\{\mathrm{LGD}_2 \cdot \mathbf{1}(\mathrm{t} < \tau_2 < \mathrm{T}) \cdot \mathrm{D}(\mathrm{t},\tau_2) \cdot [\mathrm{NPV}_0(\tau_2)]^+\right\}$$

A simplified formula without τ^{1st} for bilateral VA

- The simplified formula is only a simplified representation of bilateral risk and ignores that upon the first default closeout proceedings are started, thus involving a degree of double counting
- It is attractive because it allows for the construction of a bilateral counterparty risk pricing system based only on a unilateral one.
- The correct formula involves default dependence between the two parties through τ^{1st} and allows no such incremental construction
- A simplified bilateral formula is possible also in case of substitution closeout, but it turns out to be identical to the simplified formula of the risk free closeout case.
- We analyze the impact of default dependence between investor '0' and counterparty '2' on the difference between the two formulas by looking at a zero coupon bond and at an equity forward.

A simplified formula without τ^{1st} for bilateral VA

One can show easily that the difference between the full correct formula and the simplified formula is

$$\begin{aligned} & E_0[1_{\{\tau_0<\tau_2<T\}} LGD_2 D(0,\tau_2)(E_{\tau_2}(\Pi(\tau_2,T)))^+] \\ - \quad & E_0[1_{\{\tau_2<\tau_0<T\}} LGD_0 D(0,\tau_0)(-E_{\tau_0}(\Pi(\tau_0,T)))^+]. \end{aligned} \tag{1}$$

A simplified formula without τ^{1st}: The case of a Zero Coupon Bond

We work under deterministic interest rates. We consider $P(t, T)$ held by ‘0” (lender) who will receive the notional 1 from ‘2”(borrower) at final maturity T if there has been no default of ‘2”.

The difference between the correct bilateral formula and the simplified one is, under risk free closeout,

$$LGD_2 P(0, T)\mathbb{Q}(\tau_0 < \tau_2 < T).$$

The case with substitution closeout is instead trivial and the difference is null. For a bond, the simplified formula coincides with the full substitution closeout formula.

Therefore the difference above is the same difference between risk free closeout and substitution closeout formulas, and has been examined earlier, also in terms of contagion.

A simplified formula without τ^{1st}: The case of an Equity forward

In this case the payoff at maturity time T is given by $S_T - K$

where S_T is the price of the underlying equity at time T and K the strike price of the forward contract (typically $K = S_0$, 'at the money', or $K = S_0/P(0,T)$, 'at the money forward').

We compute the difference D^{02} between the correct bilateral risk free closeout formula and the simplified one.

A simplified formula without τ^{1st}: The case of an Equity forward

$D^{02} := A_1 - A_2$, where

$$A_1 = E_0 \left\{ 1_{\{\tau_0 < \tau_2 < T\}} LGD_2 D(0, \tau_2)(S_{\tau_2} - P(\tau_2, T)K)^+ \right\}$$
$$A_2 = E_0 \left\{ 1_{\{\tau_2 < \tau_0 < T\}} LGD_0 D(0, \tau_0)(P(\tau_0, T)K - S_{\tau_0})^+ \right\}$$

The worst cases will be the ones where the terms A_1 and A_2 do not compensate. For example assume there is a high probability that $\tau_0 < \tau_2$ and that the forward contract is deep in the money. In such case A_1 will be large and A_2 will be small.

Similarly, a case where $\tau_2 < \tau_0$ is very likely and where the forward is deep out of the money will lead to a large A_2 and to a small A_1.

However, we show with a numerical example that even when the forward is at the money the difference can be relevant. For more details see Brigo and Buescu (2011).

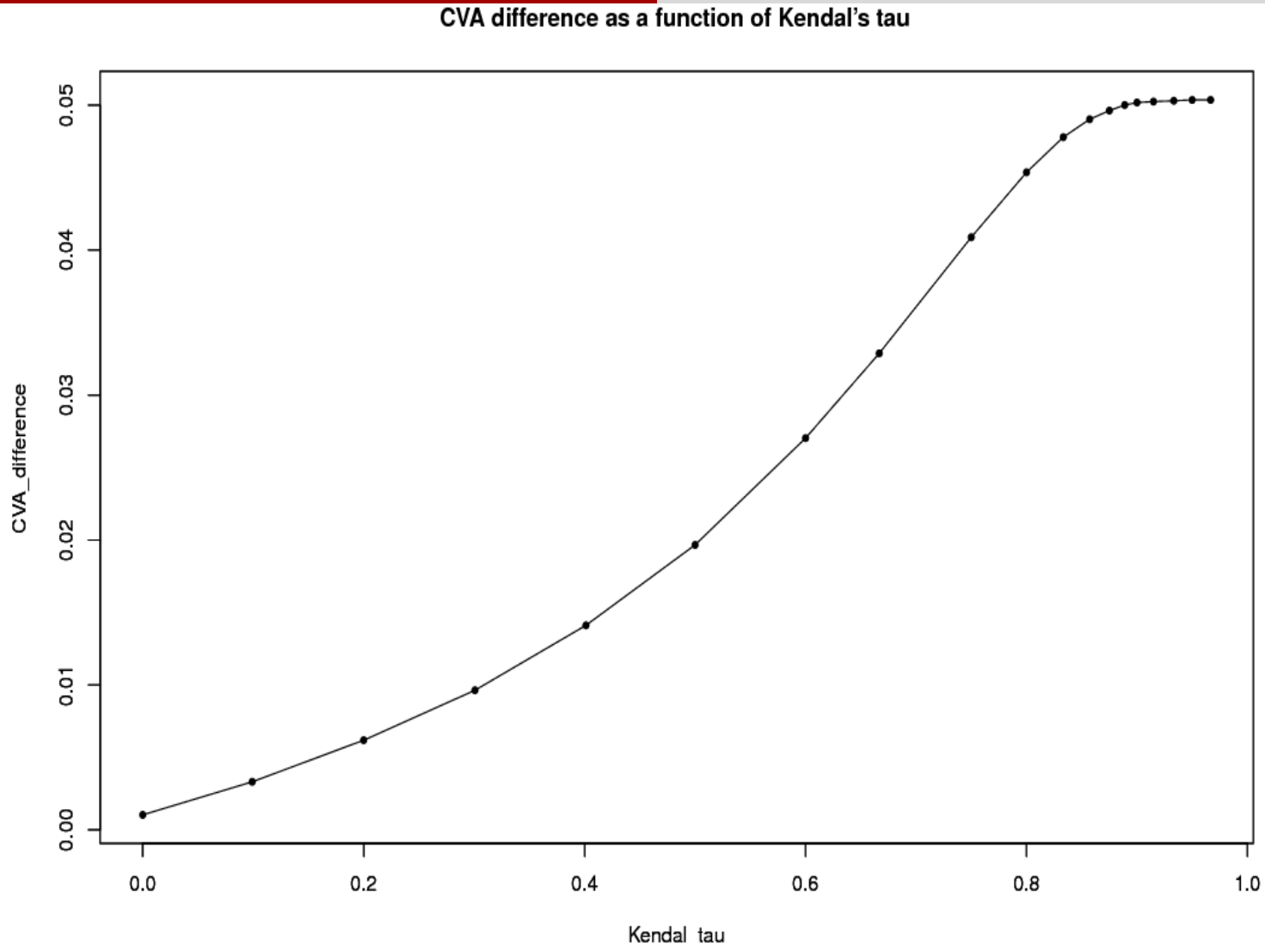


Figure: D^{02} plotted against Kendall's tau between τ_0 and τ_2, all other quantities being equal: $S_0 = 1$, $T = 5$, $\sigma = 0.4$, $K = 1$, $\lambda_0 = 0.1$, $\lambda_2 = 0.05$.

A useful derivative: Contingent CDS (CCDS)

Definition

When the reference credit defaults at τ, the protection seller pays protection on a notional that is not fixed but given by the NPV of a reference Portfolio Π at that time if positive. This amount is: $(\mathbb{E}_{\tau_C}\Pi(\tau_C, T))^+$, minus a recovery $\mathrm{R_{EC}}$ fraction of it.

CCDS default leg payoff = asymmetric counterparty risk adjustm

The payoff of the default leg of a Contingent CDS is exactly

$$(1 - \mathrm{R_{EC}})\mathbf{1}_{\{(t<\tau_C<T)\}}D(t, \tau_C)(\mathbb{E}_{\tau_C}\Pi(\tau_C, T))^+$$

General Remarks on CCDS

"[...]Rudimentary and idiosyncratic versions of these so-called CCDS have existed for five years, but they have been rarely traded due to high costs, low liquidity and limited scope. [...] Counterparty risk has become a particular concern in the markets for interest rate, currency, and commodity swaps - because these trades are not always backed by collateral.[...] Many of these institutions - such as hedge funds and companies that do not issue debt - are beyond the scope of cheaper and more liquid hedging tools such as normal CDS. The new CCDS was developed to target these institutions (Financial Times, April 10, 2008)."

Being the two payoffs equivalent, UCVA valuation will hold as well for the default leg of a CCDS.

Interest on CCDS has come back in 2011 now that CVA capital charges risk to become punitive.

Methodology

1. Assumption: The *investor* enters a transaction with a *counterparty* and, when dealing with Unilateral Risk, the investor considers itself default free.
   Note : All the payoffs seen from the point of view of the *investor*.
2. We model and calibrate the default time of the *counterparty* using a stochastic intensity default model, except in the equity case where we will use a firm value model.
3. We model the transaction underlying and estimate the deal NPV at default.
4. We allow for the counterparty default time and the contract underlying to be correlated.
5. We start however from the case when such correlation can be neglected.

Approximation: Default Bucketing

General Formulation

1. Model (underlying) to estimate the NPV of the transaction.
2. Simulations are run allowing for correlation between the credit and underlying models, to determine the counterparty default time and the underlying deal NPV respectively.

Approximated Formulation under default bucketing

$$\mathbb{E}_0\Pi^D(0,T) := \mathbb{E}_0\Pi(0,T)$$
$$-\text{LGD}\sum_{j=1}^{b}\mathbb{E}_0[1\{\tau\in(T_{j-1},T_j]\}\,D(0,T_j)(\mathbb{E}_{T_j}\Pi(T_j,T))^+]$$

1. In this formulation defaults are bucketed but we still need a joint model for τ and the underlying Π including their correlation.
2. Option model for Π is implicitly needed in τ scenarios.

Approximation: Default Bucketing and Independence

Approximated Formulation under independence (and 0 correlation)

$$\mathbb{E}_0\Pi^D(0,T) := \mathbb{E}_0\Pi(0,T)$$

$$-\text{LGD}\sum_{j=1}^{b}\boxed{\mathbb{Q}\{\tau\in(T_{j-1},T_j]\}}\mathbb{E}_0[D(0,T_j)(\mathbb{E}_{T_j}\Pi(T_j,T))^+]$$

1. In this formulation defaults are bucketed and only survival probabilities are needed (no default model).
2. Option model is STILL needed for the underlying of Π.

Ctrparty default model: CIR++ stochastic intensity

In the case where we cannot assume independence, we need an explicit default model. **The model for the counterparty instantaneous credit spread:**

$$\lambda(t) = y(t) + \psi(t; \beta)$$

Remarks:

1. $y(t)$ is a CIR process with possible jumps $dy = \kappa(\mu - y)dt + \nu\sqrt{y}dW_y + dJ$
2. $\psi(t; \beta)$ is the shift that matches a given CDS curve
3. In CDS calibration we assume deterministic interest rates.
4. Calibration : Fitting model survival probabilities to survival probabilities stripped from counterparty CDS quotes

4 cases: Interest Rates, Credit, Commodities and Equity

We now examine four specific cases of underlying contracts:

- Interest Rate Swaps and Derivatives Portfolios
- Commodities swaps (Oil)
- Credit: CDS on a reference credit
- Equity: Equity Return Swaps

Further asset classes are studied in the literature. For example see Biffis et al (2011) for CVA on longevity swaps.

Interest Rates Swap Case

Formulation for IRS under independence (no correlation)

$$\mathrm{IRS}^D(t,K) = \mathrm{IRS}(t.K)$$
$$-\mathrm{LGD}\sum_{i=a+1}^{b-1}\mathbb{Q}\{\tau\in(T_{i-1},T_i]\}\,\mathrm{SWAPTION}_{i,b}(t;K,S_{i,b}(t),\sigma_{i,b})$$

Modeling Approach with corr.

Gaussian 2-factor G2++ short-rate $r(t)$ model:
$r(t) = x(t) + z(t) + \varphi(t;\alpha), r(0) = r_0$

$dx(t) = -ax(t)dt + \sigma dW_x$
$dz(t) = -bz(t)dt + \eta dW_z$

$dW_x dW_z = \rho_{x,z}dt$

$\alpha = [r_0, a, b, \sigma, \eta, \rho_{1,2}]$

$dW_x dW_y = \rho_{x,y}dt, dW_z dW_y = \rho_{z,y}dt$

Calibration

- The function $\varphi(\cdot;\alpha)$ is deterministic and is used to calibrate the initial curve observed in the market.
- We use swaptions and zero curve data to calibrate the model.
- The r factors x and z and the intensity are taken to be correlated.

Interest Rates Swap Case

Total Correlation Counterparty default / rates

$$\bar{\rho} = \text{Corr}(dr_t, d\lambda_t) = \frac{\sigma\rho_{x,y} + \eta\rho_{z,y}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{x,z}}\sqrt{1 + \frac{2\beta\gamma^2}{\nu^2 y_t}}}.$$

where β is the intensity of arrival of λ jumps and γ is the mean of the exponentially distributed jump sizes.

Without jumps ($\beta = 0$)

$$\bar{\rho} = \text{Corr}(dr_t, d\lambda_t) = \frac{\sigma\rho_{x,y} + \eta\rho_{z,y}}{\sqrt{\sigma^2 + \eta^2 + 2\sigma\eta\rho_{x,z}}}.$$

IRS: Case Study

1) Single Interest Rate Swaps (IRS)

At-the-money fix-receiver forward interest-rate-swap (IRS) paying on the EUR market.

The IRS's fixed legs pay annually a 30E/360 strike rate, while the floating legs pay LIBOR twice per year.

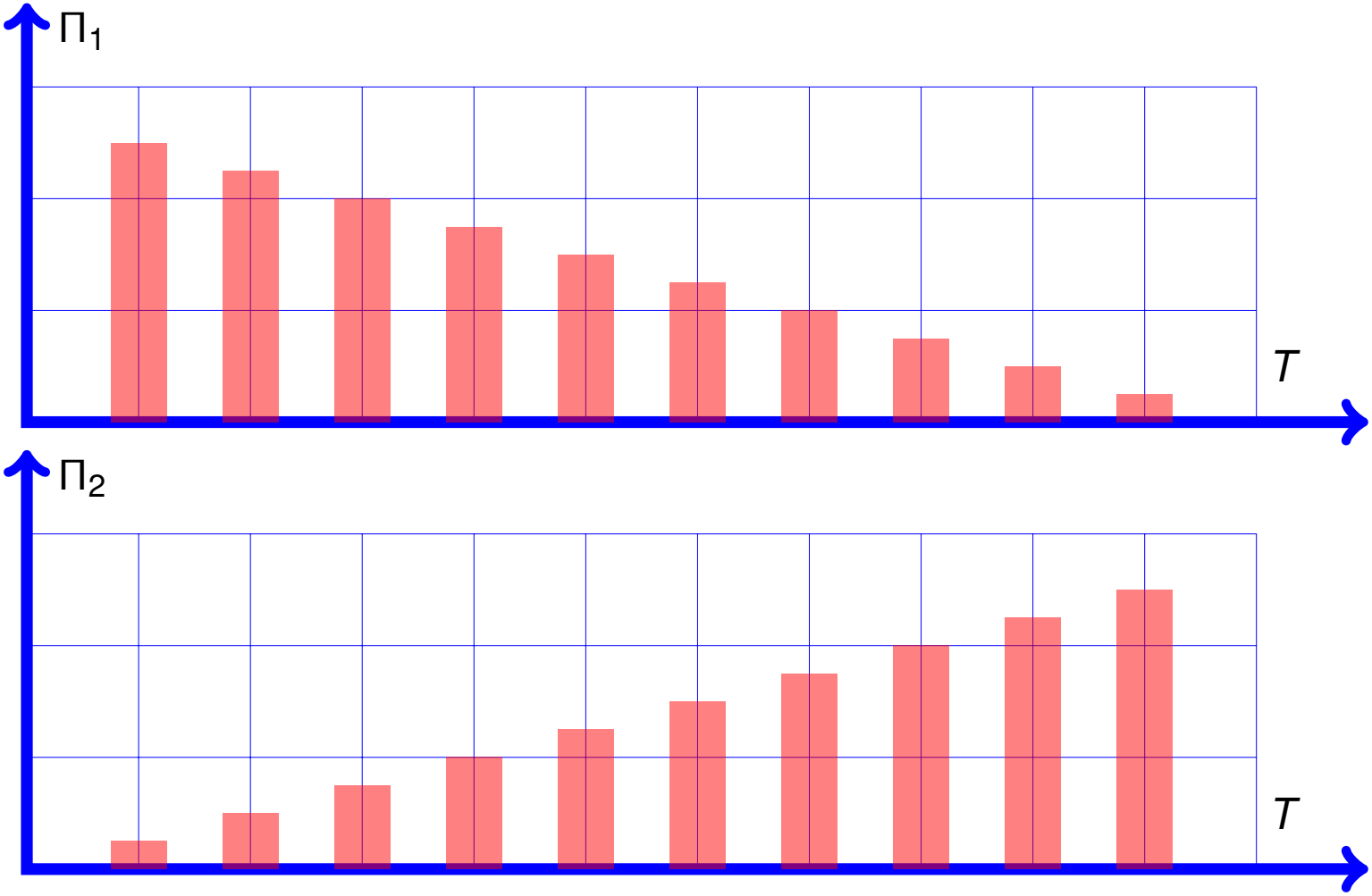
2) Netted portfolios of IRS.

- Portfolios of at-the-money IRS either with different starting dates or with different maturities.

1. ($\Pi 1$) annually spaced dates $\{T_i : i = 0 \dots N\}$, T_0 two business days from trade date; portfolio of swaps maturing at each T_i, with $i > 0$, all starting at T_0.
2. ($\Pi 2$) portfolio of swaps starting at each T_i all maturing at T_N.

Can also do exotics (Ratchets, CMS spreads, Bermudan)

IRS Case Study: Payment schedules



IRS Results

Counterparty risk price for netted receiver IRS portfolios Π1 and Π2 and simple IRS (maturity 10Y). Every IRS, constituting the portfolios, has unit notional and is at equilibrium. Prices are in bps.

λ	correlation $\bar{\rho}$	Π1	Π2	IRS
3%	-1	-140	-294	-36
	0	-84	-190	-22
	1	-47	-115	-13
5%	-1	-181	-377	-46
	0	-132	-290	-34
	1	-99	-227	-26
7%	-1	-218	-447	-54
	0	-173	-369	-44
	1	-143	-316	-37

Compare with ”Basel 2” deduced adjustments

Basel 2, under the ”Internal Model Method”, models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case, even if banks have the option to compute their own estimate of the multiplier, which can never go below 1.2 anyway.

Is this confirmed by our model?

$$(140 - 84)/84 \approx 66\% > 40\%$$

$$(54 - 44)/44 \approx 23\% < 40\%$$

So this really depends on the portfolio and on the situation.

A bilateral example and correlation risk

Finally, in the bilateral case for Receiver IRS, 10y maturity, high risk counterparty and mid risk investor, we notice that depending on the correlations

$$\bar{\rho}_0 = \mathrm{Corr}(dr_t, d\lambda_t^0), \quad \bar{\rho}_2 = \mathrm{Corr}(dr_t, d\lambda_t^2), \quad \rho_{0,2}^{Copula} = 0$$

the DVA - CVA or Bilateral CVA does change sign, and in particular for portfolios Π1 and IRS the sign of the adjustment follows the sign of the correlations.

$\bar{\rho}_2$	$\bar{\rho}_0$	Π1	Π2	10×IRS
-60%	**0%**	-117(7)	-382(12)	-237(16)
-40%	**0%**	-74(6)	-297(11)	-138(15)
-20%	**0%**	-32(6)	-210(10)	-40(14)
0%	**0%**	-1(5)	-148(9)	31(13)
20%	**0%**	24(5)	-96(9)	87(12)
40%	**0%**	44(4)	-50(8)	131(11)
60%	**0%**	57(4)	-22(7)	159(11)

Payer vs Receiver

- Counterparty Risk (CR) has a relevant impact on interest-rate payoffs prices and, in turn, correlation between interest-rates and default (intensity) has a relevant impact on the CR adjustment.
- The (positive) CR adjustment to be subtracted from the default free price **decreases with correlation for receiver payoffs**. Natural: If default intensities increase, with high positive correlation their correlated interest rates will increase more than with low correlation, and thus a receiver swaption embedded in the adjustment decreases more, reducing the adjustment.
- The adjustment for payer payoffs increases with correlation.

Further Stylized Facts

- As the default probability implied by the counterparty CDS increases, the size of the adjustment increases as well, but the impact of correlation on it decreases.
- Financially reasonable: Given large default probabilities for the counterparty, fine details on the dynamics such as the correlation with interest rates become less relevant
- **The conclusion is that we should take into account interest-rate/ default correlation in valuing CR interest-rate payoffs.**
- In the bilateral case correlation risk can cause the adjustment to change sign

Exotics

For examples on exotics, including Bermudan Swaptions and CMS spread Options, see

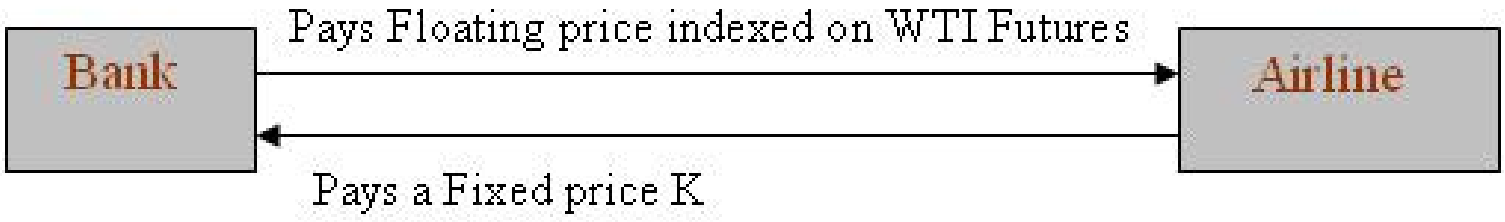
Papers with Exotics and Bilateral Risk

- Brigo, D., and Pallavicini, A. (2007). Counterparty Risk under Correlation between Default and Interest Rates. In: Miller, J., Edelman, D., and Appleby, J. (Editors), Numerical Methods for Finance, Chapman Hall.
- Brigo, D., Pallavicini, A., and Papatheodorou, V. (2009). Bilateral counterparty risk valuation for interest-rate products: impact of volatilities and correlations. Available at Defaultrisk.com, SSRN and arXiv

Commodities: Futures, Forwards and Swaps

- **Forward**: OTC contract to buy a commodity to be delivered at a maturity date T at a price specified today. The cash/commodity exchange happens at time T.
- **Future**: Listed Contract to buy a commodity to be delivered at a maturity date T. Each day between today and T margins are called and there are payments to adjust the position.
- **Commodity Swap: Oil Example**:

FIXED-FLOATING (for hedge purposes)



Commodities: Modeling Approach

Schwartz-Smith Model

$$
\begin{aligned}
ln(S_t) &= x_t + l_t + \varphi(t) \\
dx_t &= -kx_t dt + \sigma_x dW_x \\
dl_t &= \mu dt + \sigma_l dW_l \\
dW_x\ dW_l &= \rho_{x,l} dt
\end{aligned}
$$

Variables

S_t: Spot oil price;
x_t, l_t: short and long term components of S_t;
This can be re-cast in a classic convenience yield model

Correlation with credit

$$
\begin{aligned}
dW_x\ dW_y &= \rho_{x,y} dt, \\
dW_l\ dW_y &= \rho_{l,y} dt
\end{aligned}
$$

Calibration

φ: defined to exactly fit the oil forward curve.
Dynamic parameters k, μ, σ, ρ are calibrated to At the money implied volatilities on Futures options.

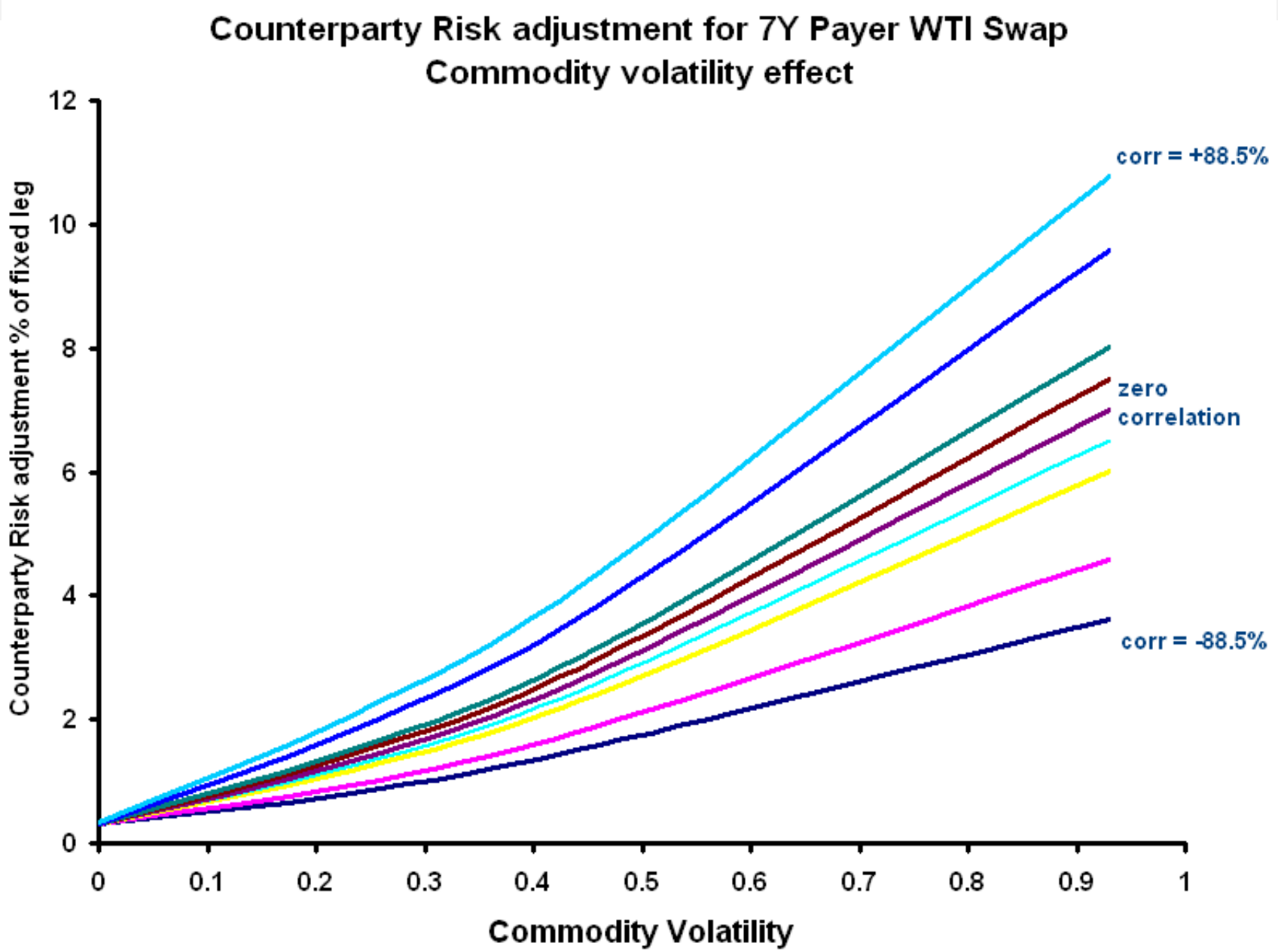
Commodities

Total correlation Commodities - Counterparty default

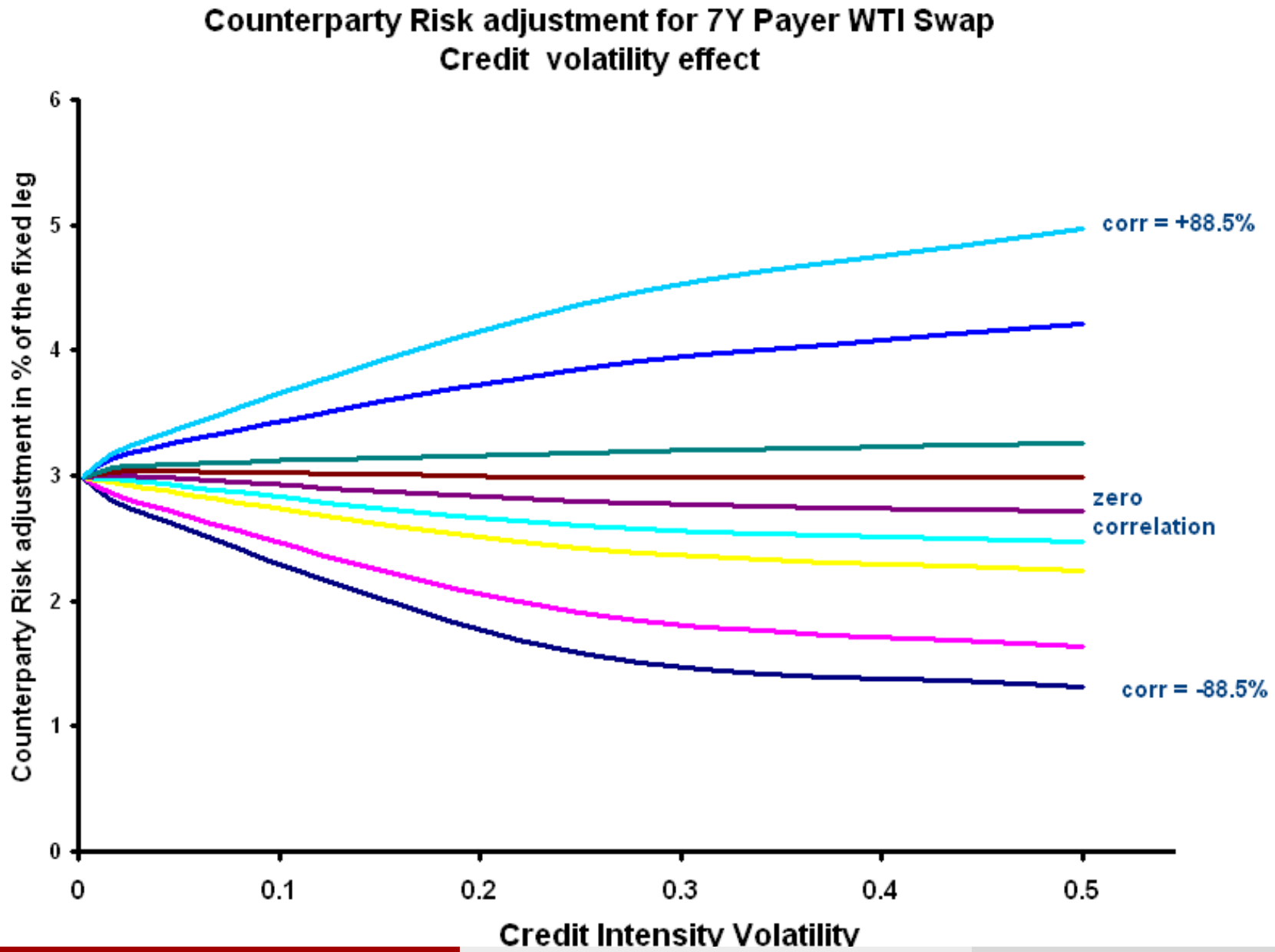
$$\bar{\rho} = \text{corr}(d\lambda_t,\ dS_t) = \frac{\sigma_x \rho_{x,y} + \sigma_L \rho_{L,y}}{\sqrt{\sigma_x^2 + \sigma_L^2 + 2\rho_{x,L}\sigma_x\sigma_L}}$$

We assumed no jumps in the intensity

Commodities: Commodity Volatility Effect



Commodities: Credit Volatility Effect



Commodities[1] : Credit volatility effect

$\bar{\rho}$	intensity volatility ν_R	0.025	0.25	0.50
-88.5	Payer adj	2.742	1.584	1.307
	Receiver adj	1.878	2.546	3.066
-63.2	Payer adj	2.813	1.902	1.63
	Receiver adj	1.858	2.282	2.632
-25.3	Payer adj	2.92	2.419	2.238
	Receiver adj	1.813	1.911	2.0242
-12.6	Payer adj	2.96	2.602	2.471
	Receiver adj	1.802	1.792	1.863
0	Payer adj	2.999	2.79	2.719
	Receiver adj	1.79	1.676	1.691
+12.6	Payer adj	3.036	2.985	2.981
	Receiver adj	1.775	1.562	1.527
+25.3	Payer adj	3.071	3.184	3.258
	Receiver adj	1.758	1.45	1.371
+63.2	Payer adj	3.184	3.852	4.205
	Receiver adj	1.717	1.154	0.977
+88.5	Payer adj	3.229	4.368	4.973
	Receiver adj	1.664	0.988	0.798

Fixed Leg Price maturity 7Y: 7345.39 USD for a notional of 1 Barrel per Month

[1] adjusment expressed as % of the fixed leg price

Commodities[2] : Commodity volatility effect

$\bar{\rho}$	Commodity spot volatility σ_S	0.0005	0.232	0.46	0.93
-88.5	Payer adj	0.322	0.795	1.584	3.607
	Receiver adj	0	1.268	2.546	4.495
-63.2	Payer adj	0.322	0.94	1.902	4.577
	Receiver adj	0	1.165	2.282	4.137
-25.3	Payer adj	0.323	1.164	2.419	6.015
	Receiver adj	0	0.977	1.911	3.527
-12.6	Payer adj	0.323	1.246	2.602	6.508
	Receiver adj	0	0.917	1.792	3.325
0	Payer adj	0.324	1.332	2.79	6.999
	Receiver adj	0	0.857	1.676	3.115
+12.6	Payer adj	0.324	1.422	2.985	7.501
	Receiver adj	0	0.799	1.562	2.907
+25.3	Payer adj	0.324	1.516	3.184	8.011
	Receiver adj	0	0.742	1.45	2.702
+63.2	Payer adj	0.325	1.818	3.8525	9.581
	Receiver adj	0	0.573	1.154	2.107
+88.5	Payer adj	0.326	2.05	4.368	10.771
	Receiver adj	0	0.457	0.988	1.715

Fixed Leg Price maturity 7Y: 7345.39 USD for a notional of 1 Barrel per Month

[2] adjusment expressed as % of the fixed leg price

Wrong Way Risk?

Basel 2, under the ”Internal Model Method”, models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case, even if banks have the option to compute their own estimate of the multiplier, which can never go below 1.2 anyway.
What did we get in our cases? Two examples:

$$(4.973 - 2.719)/2.719 = 82\% >> 40\%$$

$$(1.878 - 1.79)/1.79 \approx 5\% << 20\%$$

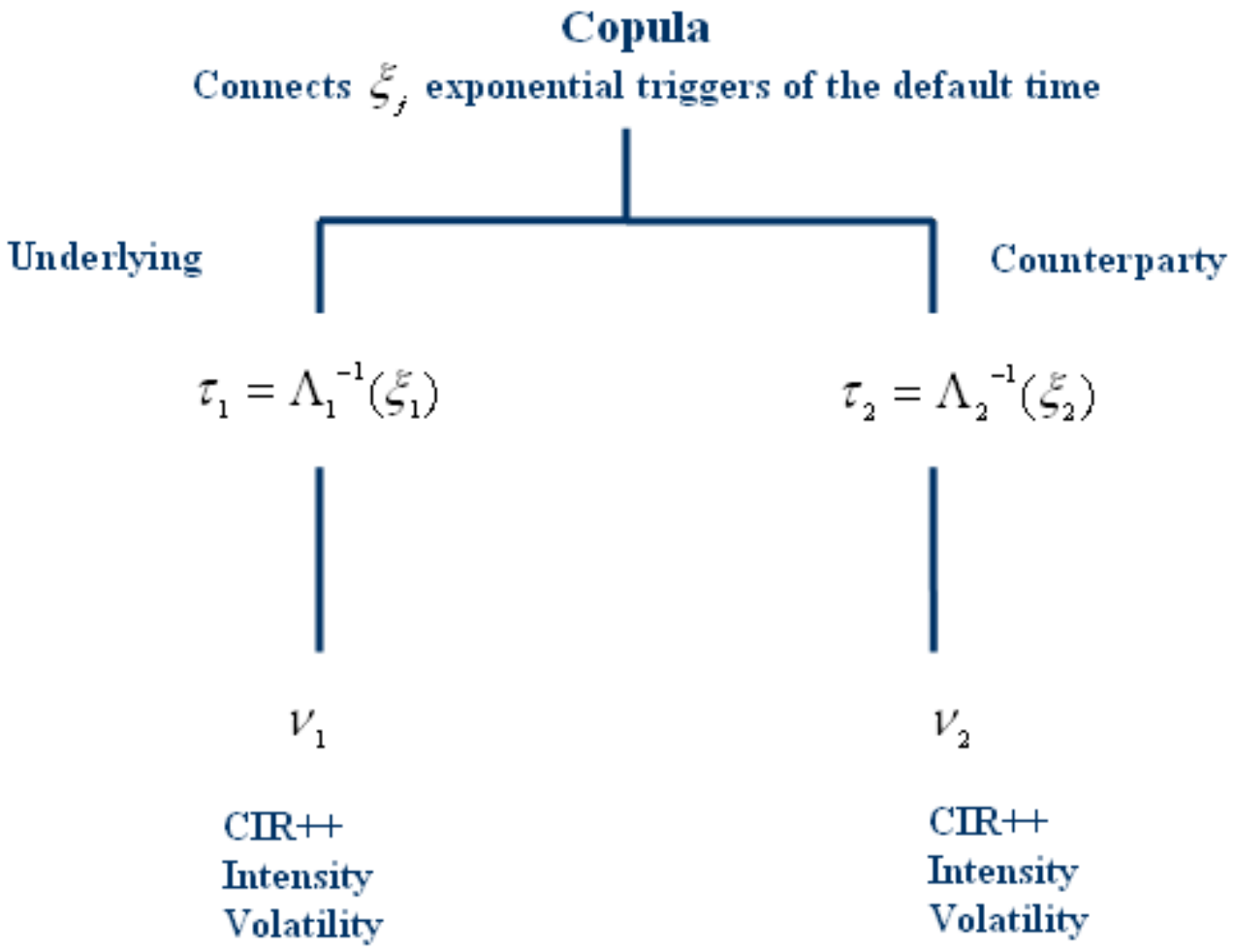
Credit (CDS)

- Model equations: (”1” = CDS underlying, ”2” = counterparty)

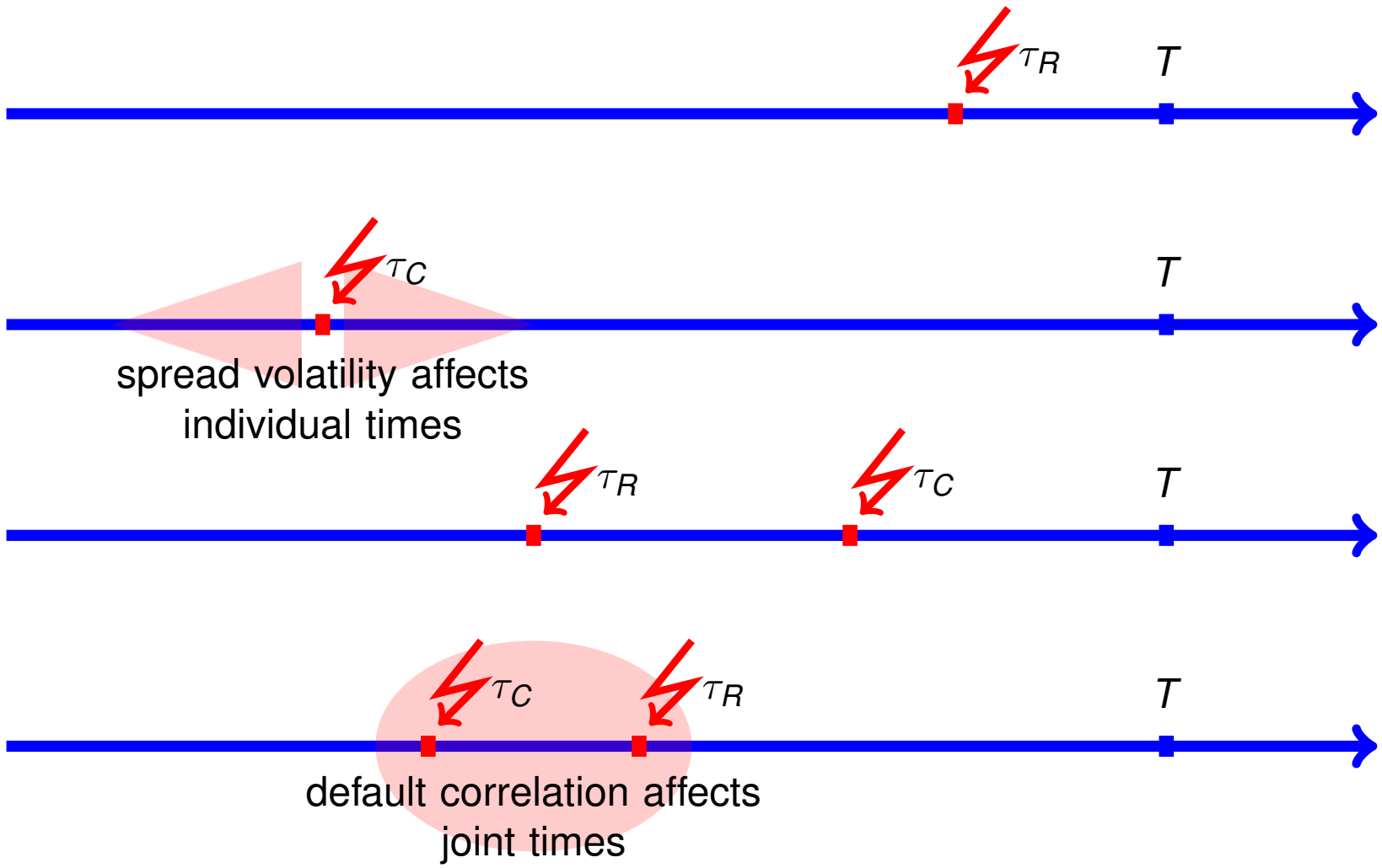
$$d\lambda_j(t) = k_j(\mu_j - \lambda_j(t))dt + \nu_j\sqrt{\lambda_j(t)}dZ_j(t), \quad j = 1, 2$$

- Cumulative intensities are defined as : $\Lambda(t) = \int_0^t \lambda(s)ds$.
- Default times are $\tau_j = \Lambda_j^{-1}(\xi_j)$. Exponential triggers ξ_1 and ξ_2 are connected through a gaussian copula with correlation parameter ρ.
- In our approach, we take into account default correlation between default times τ_1 and τ_2 **and** credit spreads volatility $\nu_j, j = 1, 2$.
- Important: volatility can amplify default time uncertainty, while high correlation reduces conditional default time uncertainty.
  Taking into account ρ and $\nu \Longrightarrow$ better representation of market information and behavior, especially for wrong way risk.

Credit (CDS) : Overview



Credit (CDS) Correlation and Volatility Effects

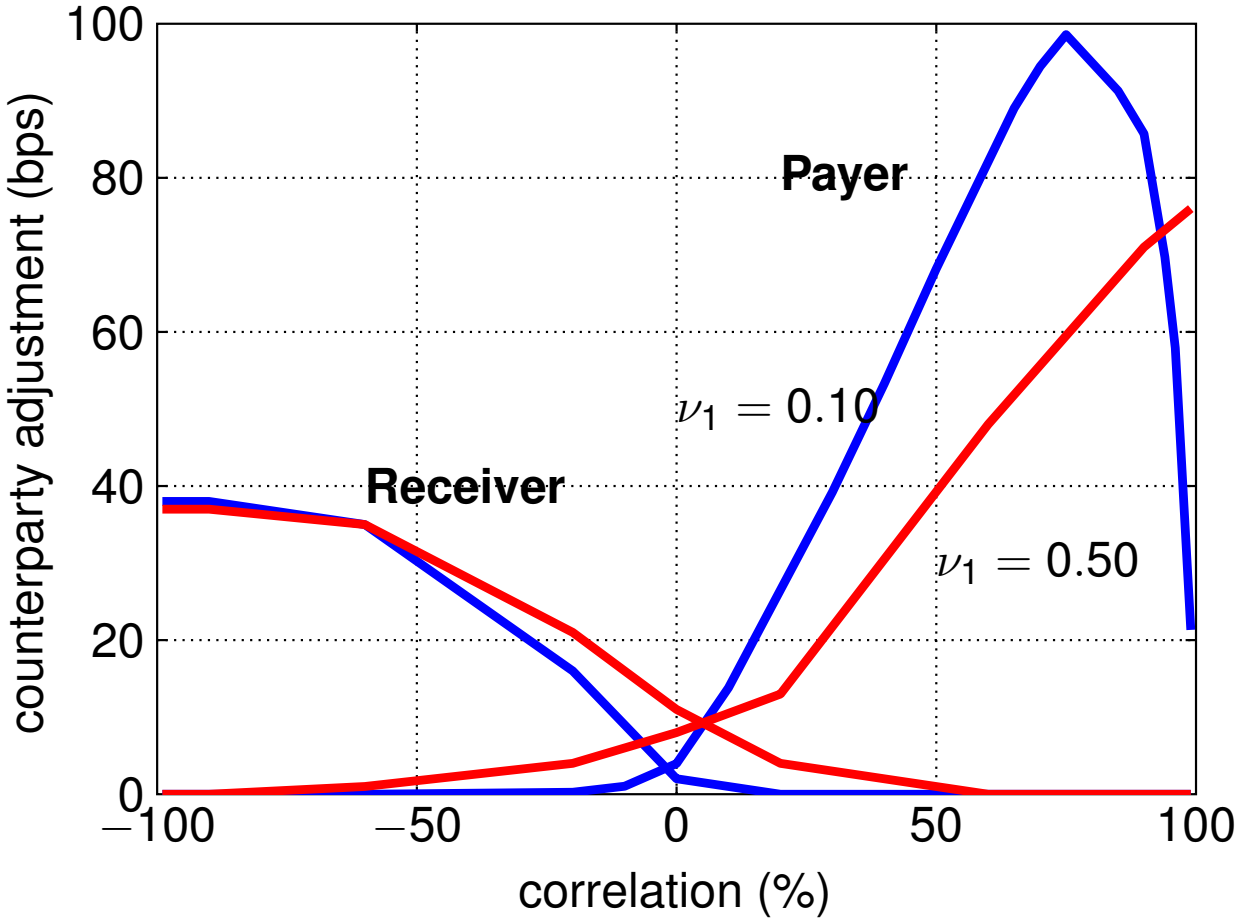


Moderate counterparty spread $\nu_2 = 0.10$

ρ	Vol parameter ν_1	0.01	0.10	0.20	0.30	0.40	0.50
	CDS Implied vol	1.5%	15%	28%	37%	42%	42%
-99	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	40(2)	38(2)	39(2)	38(2)	36(1)	37(1)
-90	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	39(2)	38(2)	38(2)	38(2)	35(1)	37(2)
-60	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	1(0)
	Receiver adj	36(1)	35(1)	36(1)	36(1)	32(1)	35(1)
-20	Payer adj	0(0)	0(0)	1(0)	2(0)	3(0)	4(1)
	Receiver adj	16(1)	16(1)	17(1)	19(1)	18(1)	21(1)
0	Payer adj	3(0)	4(0)	5(0)	7(1)	7(1)	8(1)
	Receiver adj	0(0)	2(0)	5(0)	8(0)	10(0)	11(1)
+20	Payer adj	27(1)	25(1)	23(1)	20(1)	16(2)	13(1)
	Receiver adj	0(0)	0(0)	1(0)	2(0)	2(0)	4(0)
+60	Payer adj	80(4)	82(4)	67(4)	64(4)	55(3)	48(3)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
+90	Payer adj	87(6)	86(6)	88(6)	78(5)	80(5)	71(4)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
+99	Payer adj	10(2)	21(3)	52(5)	68(5)	73(5)	76(5)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)

Large counterparty spread $\nu_2 = 0.20$

ρ	Vol parameter ν_1	0.01	0.10	0.20	0.30	0.40	0.50
	CDS Implied vol	1.5%	15%	28%	37%	42%	42%
-99	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	41(2)	40(2)	39(2)	40(2)	40(2)	40(2)
-90	Payer adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
	Receiver adj	41(2)	39(2)	39(2)	41(2)	40(2)	40(2)
-60	Payer adj	0(0)	0(0)	0(0)	0(0)	1(0)	1(0)
	Receiver adj	39(1)	37(1)	37(1)	37(1)	36(1)	35(1)
-20	Payer adj	0(0)	0(0)	2(0)	3(0)	3(0)	4(1)
	Receiver adj	17(1)	17(1)	17(1)	19(1)	21(1)	20(1)
0	Payer adj	3(0)	5(0)	6(0)	7(1)	6(1)	6(1)
	Receiver adj	0(0)	2(0)	4(0)	7(0)	10(0)	12(1)
+20	Payer adj	25(1)	24(1)	23(1)	20(1)	17(1)	15(1)
	Receiver adj	0(0)	0(0)	1(0)	2(0)	2(0)	4(0)
+60	Payer adj	74(4)	74(4)	69(4)	59(3)	54(3)	52(3)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	1(0)
+90	Payer adj	91(6)	90(6)	88(5)	80(5)	81(5)	81(5)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)
+99	Payer adj	43(4)	56(5)	57(5)	72(5)	74(5)	78(5)
	Receiver adj	0(0)	0(0)	0(0)	0(0)	0(0)	0(0)


100
80
60
40
20
0
−100
−50
0
50
100
counterparty adjustment (bps)
correlation (%)
Payer
Receiver
$\nu_1 = 0.10$
$\nu_1 = 0.50$

CDS: Bilateral

- Need to add one Model equations: (”0” = CDS investor)

$$d\lambda_j(t) = k_j(\mu_j - \lambda_j(t))dt + \nu_j\sqrt{\lambda_j(t)}dZ_j(t), \quad j = 0, 1, 2$$

- Cumulative intensities are defined as : $\Lambda(t) = \int_0^t \lambda(s)ds$.
- Default times are $\tau_j = \Lambda_j^{-1}(\xi_j)$. Exponential triggers ξ_0, ξ_1 and ξ_2 are connected through a gaussian copula with correlation parameters r_{01}, r_{02} and r_{12}.
- In our approach, we take into account default correlation between default times τ_0, τ_1 and τ_2 **and** credit spreads volatility $\nu_j, j = 0, 1, 2$.
- Important: volatility can amplify default time uncertainty, while high correlation reduces conditional default time uncertainty.
  Taking into account vols and correlations $\Longrightarrow$ better representation of market information and behavior, especially for wrong way risk.

CDS: Bilateral Adjustment to be subtracted

CDS on reference entity "1" traded between investor "0" (protection seller) and counterparty "2" (protection buyer). $\tau = \min(\tau_0, \tau_2)$.

$$
\begin{aligned}
&\mathrm{CVA}_t - \mathrm{DVA}_t = \\
&= LGD_2 \cdot \mathbb{E}_t \left\{ \mathbf{1}_{\tau=\tau_2 \leq T} \cdot D(t, \tau_2) \cdot \left[\mathbf{1}_{\tau_1 > \tau_2} \overline{\mathrm{CDS}}_{a,b}(\tau_2, S, LGD_1) \right]^+ \right\} \\
&- LGD_0 \cdot \mathbb{E}_t \left\{ \mathbf{1}_{\tau=\tau_0 \leq T} \cdot D(t, \tau_0) \cdot \left[-\mathbf{1}_{\tau_1 > \tau_0} \overline{\mathrm{CDS}}_{a,b}(\tau_0, S, LGD_1) \right]^+ \right\}
\end{aligned}
$$

CDS: Bilateral Adjustment to be subtracted

where

$$
\begin{aligned}
& \overline{\mathrm{CDS}}_{a,b}(T_j, S_1, LGD_1) = \\
:= \; & \Bigg\{ S_1 \Bigg[- \int_{\max\{T_a, T_j\}}^{T_b} D(T_j, t)(t - T_{\gamma(t)-1}) d\mathbb{Q}(\tau_1 > t | \mathcal{G}_{T_j}) \\
& + \sum_{i=\max\{a,j\}+1}^{b} \alpha_i D(T_j, T_i) \mathbb{Q}(\tau_1 > T_i | \mathcal{G}_{T_j}) \Bigg] \\
& + \mathrm{L}_{\mathrm{GD}_1} \Bigg[\int_{\max\{T_a, T_j\}}^{T_b} D(T_j, t) d\mathbb{Q}(\tau_1 > t | \mathcal{G}_{T_j}) \Bigg] \Bigg\}
\end{aligned}
$$

Key quantities are CONDITIONAL default probabilities. Conditioning makes their calculations quite complicated, bringing in a number of non-tractable copula terms. We used Fourier transforms techniques combined with copula computations.

CDS Bilateral Adjustment: A market case with Lehman, Shell and BA

Maturity	Royal Dutch Shell	Lehman Brothers	British Airways
1y	4/24	6.8/203	10/151
2y	5.8/24.6	10.2/188.5	23.2/230
3y	7.8/26.4	14.4/166.75	50.6/275
4y	10.1/28.5	18.7/152.25	80.2/305
5y	11.7/30	23.2/145	110/335
6y	15.8/32.1	27.3.3/136.3	129.5/342
7y	19.4/33.6	30.5/130	142.8/347
8y	20.5/35.1	33.7/125.8	153.6/350.6
9y	21/36.3	36.5/122.6	162.1/353.3
10y	21.4/37.2	38.6/120	168.8/355.5

Market spread quotes in basis points for Royal Dutch Shell, Lehman Brothers and British Airways on January 5, 2006 and May 1, 2008. The notation a/c indicates that a is the CDS spread on T_a =Jan 5, 2006, while c is the CDS spread on T_c = May 1, 2008.

CDS Bilateral Adjustment: volatility dynamics

2006/2008	$y(0)$	κ	μ	ν
Lehm "0"	0.0001/ 0.6611	0.036/ 7.8788	0.0432/ 0.0208	0.0553/ 0.5722
Shell "1"	0.0001/0.003	0.0394/0.1835	0.0219/0.0089	0.0192/ 0.0057
BA "2"	0.00002/0.00001	0.0266/0.6773	0.2582/0.0782	0.0003/ 0.2242

The CIR parameters of Lehman Brothers, Royal Dutch Shell and British Airways calibrated to the market quotes of CDS on January 5, 2006, and May 1, 2008. The notation a/c indicates that a is the CIR parameter on $T_a =$Jan 5 2006, while c is the CIR parameter on $T_c =$May 1, 2008.

CDS Bilateral Adjustment: MTM pre- / in- crisis

We calculate the mtm of the CDS contract as follows:

$$MTM_{a,c}(S_1, LGD_{012}) = CDS^D_{c,d}(T_c, S_1, LGD) - \frac{CDS^D_{a,b}(T_a, S_1, LGD)}{D(T_a, T_c)}$$

CDS Bilateral Adjustment: MTM pre- / in- crisis

r_{01} r_{02} r_{12}	**Vol. ν_1** **CDS IV**	**0.01** **1.5%**	**0.10** **15%**	**0.20** **28%**	**0.30** **37%**	**0.40** **42%**	**0.50** **42%**
-.3, -.3, .6	L Pay, BA R BA P, L Rec	39.1(2.1) -84.2(0.0)	44.7(2.0) -83.8(0.1)	51.1(1.9) -83.5(0.1)	58.4(1.4) -83.8(0.1)	60.3(1.7) -83.8(0.2)	63.8(1.1) -83.8(0.2)
-.3, -.3, .8	L P, BA R BA P, L R	13.6(3.6) -84.2(0.0)	22.6(3.2) -83.9(0.1)	35.2(2.6) -83.6(0.1)	43.7(2.0) -83.9(0.1)	45.3(2.4) -83.9(0.2)	52.0(1.4) -83.8(0.2)
.6, -.3, -.2	L P, BA R BA P, L R	83.1(0.0) -55.6(1.8)	81.9(0.2) -58.7(1.7)	81.6(0.3) -66.1(1.4)	82.4(0.3) -71.3(1.1)	82.6(0.3) -73.2(1.0)	82.8(0.4) -74.1(0.9)
.8, -.3, -.3	L P, BA R BA P, L R	83.9(0.0) -36.4(3.3)	82.9(0.1) -41.9(3.0)	82.3(0.3) -55.9(2.2)	82.9(0.2) -63.4(1.6)	82.9(0.3) -65.8(1.5)	83.0(0.3) -66.4(1.5)
0, 0, .5	L P, BA R BA P, L R	50.6(1.5) -80.9(0.2)	54.3(1.5) -80.5(0.3)	59.2(1.5) -80.9(0.4)	64.4(1.1) -82.3(0.3)	65.5(1.3) -82.6(0.3)	68.8(0.8) -82.8(0.3)
0, 0, .8	L P, BA R BA P, L R	12.3(3.5) -80.9(0.2)	21.0(3.0) -81.5(0.2)	34.9(2.5) -81.9(0.3)	41.3(2.1) -81.9(0.4)	44.6(1.9) -82.1(0.4)	50.6(1.4) -82.7(0.3)
0, 0, 0	L P, BA R BA P, L R	78.1(0.2) -81.6(0.2)	77.9(0.3) -81.9(0.2)	79.5(0.5) -82.3(0.3)	79.5(0.5) -82.2(0.4)	80.1(0.6) -82.7(0.3)	82.1(0.4) -83.2(0.3)
0, .7, 0	L P, BA R BA P, L R	77.3(0.3) -81.2(0.2)	77.3(0.4) -81.8(0.2)	78.5(0.5) -81.9(0.3)	79.2(0.5) -80.8(1.3)	79.7(0.6) -82.4(0.3)	81.5(0.4) -82.6(0.3)
.3, .2, .6	L P, BA R BA P, L R	54.1(1.4) -81.3(0.2)	56.7(1.3) -81.7(0.2)	62.5(1.1) -81.4(0.4)	63.6(1.1) -81.3(0.5)	66.4(0.9) -81.6(0.4)	69.7(0.6) -82.1(0.4)
.3, .3, .8	L P, BA R BA P, L R	22.8(4.2) -83.0(0.2)	28.8(3.5) -83.2(0.2)	38.6(2.9) -82.8(0.3)	42.6(2.9) -82.4(0.4)	45.9(2.5) -82.5(0.4)	52.0(2.2) -82.9(0.4)
.5, .5, .5	L P, BA R BA P, L R	62.8(0.8) -67.4(1.1)	64.5(0.8) -70.4(0.9)	67.7(0.8) -72.9(0.9)	68.5(0.9) -74.4(0.9)	71.3(0.7) -75.8(0.8)	73.2(0.6) -76.7(0.7)
.7, 0, 0	L P, BA R BA P, L R	77.4(0.2) -47.3(2.2)	77.3(0.3) -55.0(1.9)	78.9(0.5) -61.6(1.6)	79.1(0.5) -65.0(1.5)	79.9(0.5) -67.5(1.3)	81.4(0.4) -69.6(1.1)

CDS marked to market by Lehman Brothers on May 1, 2008. The mark-to-market value of the CDS without risk adjustment when Lehman Brothers is respectively payer (receiver) is 84.2(-84.2) bps, due to the widening of Shell spreads.

CDS Bilateral Adjustment: MTM pre- / in- crisis

Negative or null BA“2”/Shell“1” dependence

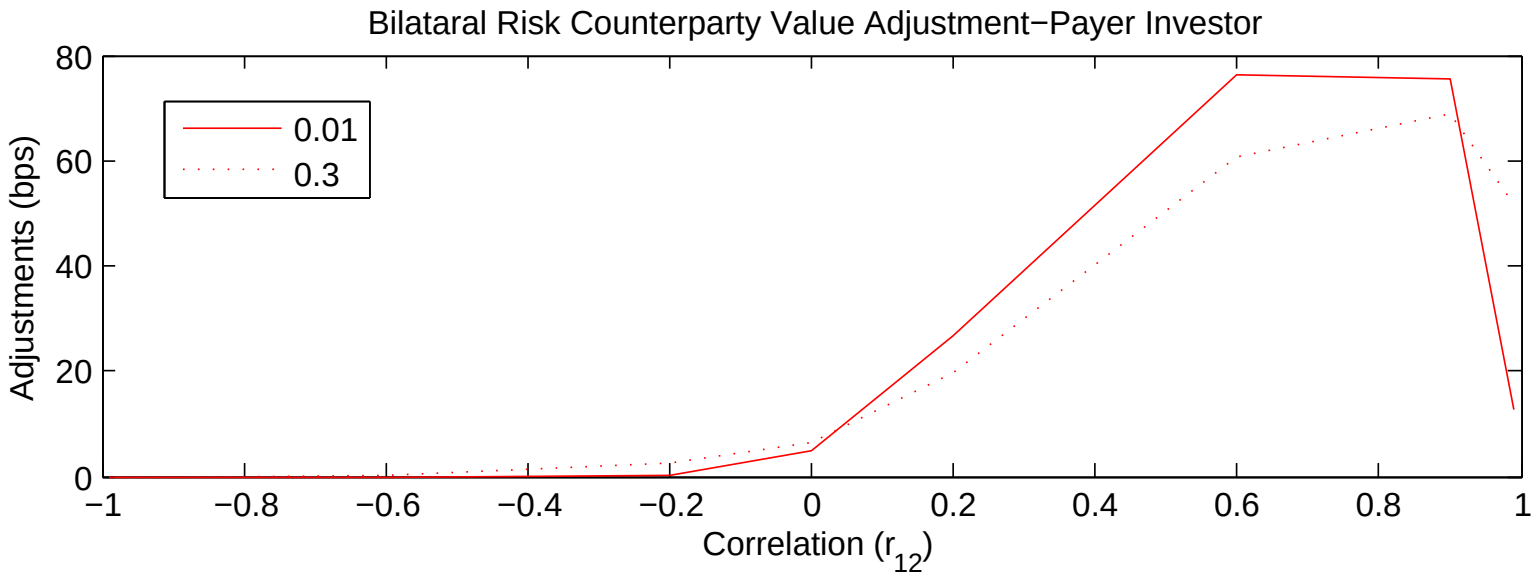
- If BA is negatively correlated or uncorrelated to Shell, see triples $(0.6, -0.3, -0.2)$, $(0.8, -0.3, -0.3)$, and $(0.7, 0, 0)$, then the MTM appears to be the largest for Lehman
- This happens whether Lehman is the CDS payer or the CDS receiver.

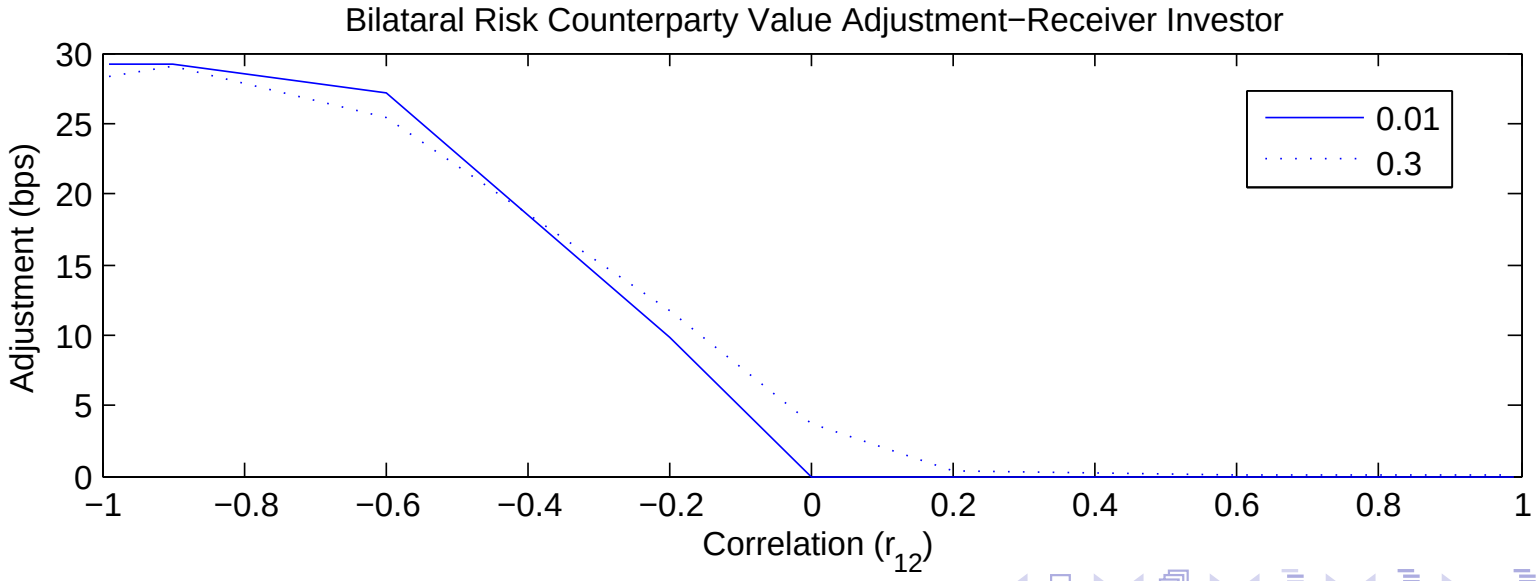
CDS Bilateral Adjustment: MTM pre- / in- crisis

Shell“1” credit spread volatility

- Increases in credit spreads volatility of Shell increase the MTM when Lehman is the CDS payer and decrease the contract valuation when Lehman is the CDS receiver.
- Conversely, if Lehman is receiver, this implies smaller CDS contract valuations for Lehman.

CDS Bilateral on a different portfolio: Wrong way risk





Equity: Intensity vs Firm value models

If we have equity S_t of a name '1' as contract underlying and we have the default of the counterparty

$$\tau_2 = \Lambda_2^{-1}(\xi_2)$$

it's hard to correlate τ_2 and S_1 enough, given that the exponential random variable ξ_2 and any Brownian motion W_1 driving S_1 will necessarily be independent.

Underlying Equity/ Counterparty Default correlation

The only hope to create correlation is to put a stochastic λ_2 and correlate it with W_1 driving S_1. However, since most of the randomness of τ_2 comes from ξ_2, this does not create enough correlation.

With equity we change family of credit models, and resort to Firm Value (or structural) models for the default of the counterparty.

Equity: Intensity vs Firm Value models

Intensity VS Firm Value models

$$\tau_2 = \Lambda_2^{-1}(\xi_2) \quad \text{vs} \quad \tau_2 = \inf\{t : V(t) \leq H(t)\}$$

Default of the counterparty is the first time when the counterparty firm value V hits a default barrier H.

Equity/Credit Correlation with Firm Value Models

Now if the underlying equity S_1 is driven by a brownian motion W_1,

$$dS_1(t) = (r - y_1)S_1(t)dt + \sigma_1(t)S_1(t)dW_1(t)$$

and the counterparty V is also driven by a brownian motion W_2,

$$dV(t) = (r - q)V(t)dt + \sigma(t)V(t)dW_2(t)$$

then an effective way to create correlation is $dW_1 dW_2 = \rho_{12} dt$

Equity: Firm Value models for the counterparty default

AT1P model

Let the risk neutral firm value V dynamics and the default barrier $\widehat{H}(t)$ of the counterparty '2' be

$$dV(t) = V(t)(r(t) - q(t))dt + V(t)\sigma(t)dW(t)$$

$$H(t) = \frac{H}{V_0}\mathbb{E}\left[V_t\right]\ e^{\left(-B\int_0^t \sigma_s^2 ds\right)}$$

and let the default time τ be **the 1st time** V **hits** $H(t)$ **from above**, starting from $V_0 > H$. Here $H > 0$ and B are free parameters we may use to shape the barrier.

Then the survival probability is given analytically in close form by a barrier option type formula (see Brigo and Tarenghi (2005) and Brigo, Morini and Tarenghi (2011)).

Firm Value model Calibration to CDS data

It is possible to fit exactly the CDS spreads for the counterparty through the firm value volatility $\sigma(t)$ using a bootstrapping procedure.

$$\left.\begin{array}{c} S^{\text{MktCDS}}_{0,1y} \\ S^{\text{MktCDS}}_{0,2y} \\ \vdots \\ S^{\text{MktCDS}}_{0,10y} \end{array}\right\} \longleftrightarrow \left\{\begin{array}{c} dV(t) = (r-q)V(t)dt + \sigma_V(t)V(t)dW(t) \\ H(t) \\ \text{model parameters: } \sigma_V(t) \end{array}\right.$$

This ensures that the firm value model is consistent with liquid credit data of the counterparty.
In the papers we give examples based on Lehman and Parmalat.

Counterparty risk in equity return swap (ERS)

$$
\begin{array}{lcccc}
\text{Initial Time 0:} & & \text{NO FLOWS, or} & & \\
\boxed{0} & \longrightarrow & KS_0 \text{ cash} & \longrightarrow & \boxed{2} \\
 & \longleftarrow & K \text{ equity} & \longleftarrow & \\
 & & \ldots. & & \\
\text{Time } T_i: & \longrightarrow & \text{equity dividends} & \longrightarrow & \\
\boxed{0} & \longleftarrow & \text{Libor + Spread} & \longleftarrow & \boxed{2} \\
 & & \ldots. & & \\
\text{Final Time } T_b: & \longrightarrow & K \text{ equity or } KS_{T_b} \text{ cash} & \longrightarrow & \\
\boxed{0} & \longleftarrow & KS_0 \text{ cash} & \longleftarrow & \boxed{2}
\end{array}
\tag{2}
$$

Counterparty risk in equity return swap (ERS)

- We are a default-free company "0" entering a contract with counterparty "2". The reference underlying equity is "1".
- "0" and "2" agree on an amount K of stocks of "C" (with price S) to be taken as nominal ($N = K\,S_0$). The contract starts in $T_a = 0$ and has final maturity $T_b = T$.
- At $t = 0$ there is no exchange of cash (alternatively, we can think that "B" delivers to "A" an amount K of "C" stock and receives a cash amount equal to KS_0).
- At intermediate times "A" pays to "B" the dividend flows of the stocks (if any) in exchange for a periodic risk free rate plus a spread X.
- At final maturity $T = T_b$, "A" pays KS_T to "B" (or gives back the amount K of stocks) and receives a payment KS_0.

The (fair) spread X is chosen in order to obtain a contract whose value at inception is zero.

Counterparty risk in equity return swap (ERS)

$S_0 = 20$, volatility $\sigma = 20\%$ and constant dividend yield $y = 0.80\%$. The simulation date is September 16th, 2009. The contract has maturity $T = 5y$ and the settlement of the risk free rate has a semi-annual frequency. Finally, we included a recovery rate $\text{R}_{\text{EC}} = 40\%$ for the counterparty default.

T_i	$S_i^{BID,CDS}$ (bps)	$S_i^{ASK,CDS}$ (bps)
1y	25	31
3y	34	39
5y	42	47
7y	46	51
10y	50	55

Table: CDS spreads used for the counterparty "B" credit quality in the valuation of the equity return swap.

Counterparty risk in equity return swap (ERS)

Fair spread X is driven by CVA

We compute the unilateral CVA adjustment by simulation in the model above. We search for the spread X such that the total value of the ERS INLCUDING THE CVA ADJUSTMENT is zero. In fact, it can be proven that without counterparty credit risk the theoretical fair spread X would be 0. We see that the spread X is due entirely to counterparty risk.

ρ	X (AT1P)
-1	0.0
-0.2	3.0
0	5.5
0.5	14.7
1	24.9

Table: Fair spread X (in basis points) of the Equity Return Swap in five different correlation cases for AT1P.

Compare with ”Basel 2” deduced adjustments

Basel 2, under the ”Internal Model Method”, models wrong way risk by means of a 1.4 multiplying factor to be applied to the zero correlation case, even if banks have the option to compute their own estimate of the multiplier, which can never go below 1.2 anyway.

Is this confirmed by our model?

$$(24.9 - 5.5)/5.5 \approx 353\% >> 40\%$$

Collateral Management and Gap Risk I

- Risk-neutral evaluation of counterparty risk in presence of collateral management can be a difficult task, due to the complexity of clauses.
- Only few papers in the literature deal with it. Among them we cite Cherubini (2005), Alavian *et al.* (2008), Yi (2009), Assefa *et al.* (2009), Brigo et al (2011) and citations therein.

Collateral Management and Gap Risk II

- Collateralized bilateral CVA for a netted portfolio of IRS with ten year maturity and one year coupon tenor for different default-time correlations with (and without) collateral re-hypothecation. See Brigo, Capponi, Pallavicini and Papatheodorou (2010), forthcoming, from which this presentation is taken.

Collateral Management and Gap Risk III

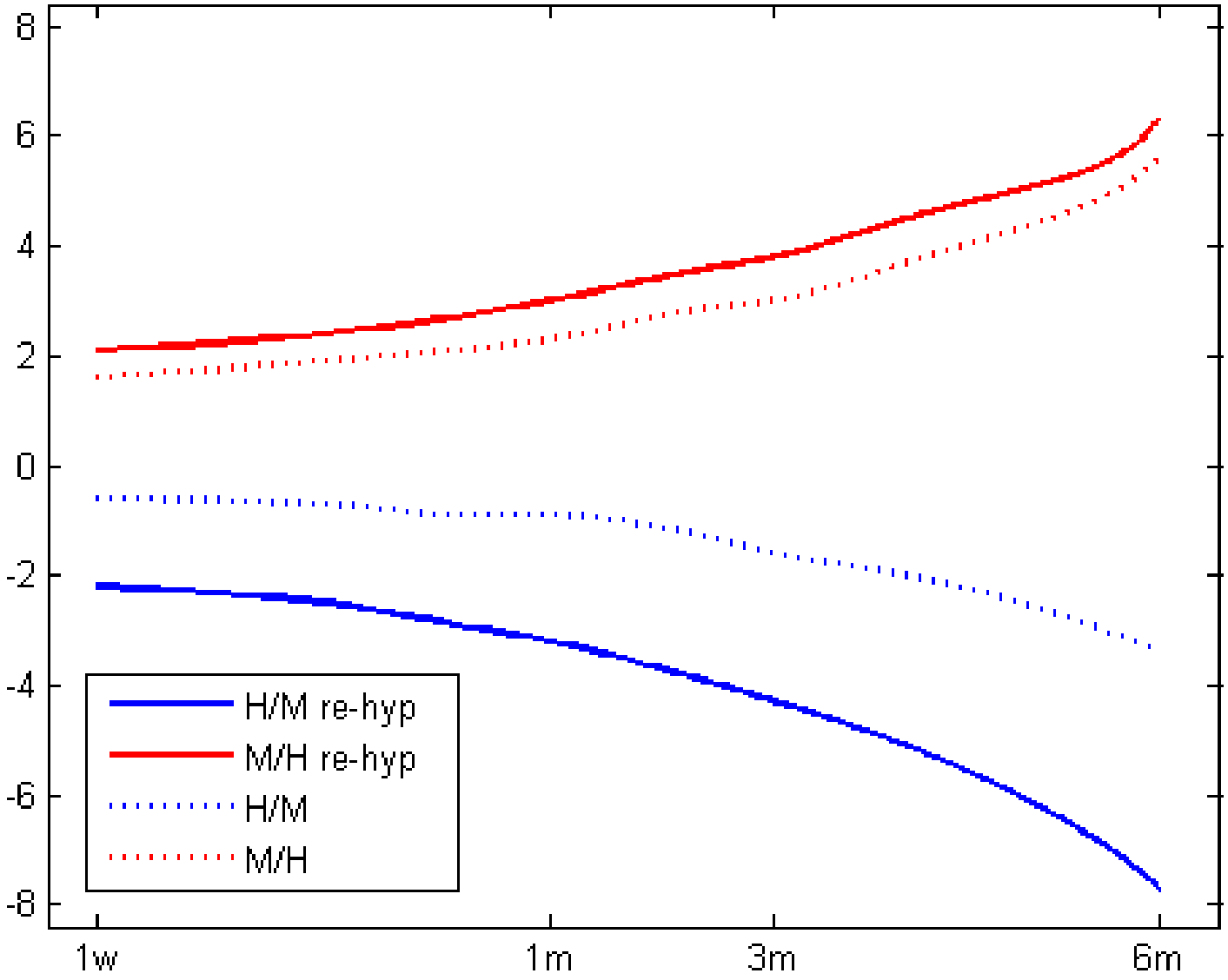


Figure explanation

Bilateral valuation adjustment, margining and rehypotecation

The figure shows the BVA(DVA-CVA) for a ten-year IRS under collateralization through margining as a function of the update frequency δ with zero correlation between rates and counterparty spread, zero correlation between rates and investor spread, and zero correlation between the counterparty and the investor defaults. The model allows for nonzero correlations as well.
Continuous lines represent the re-hypothecation case, while **dotted lines** represent the opposite case. The *red line* represents an investor riskier than the counterparty, while the *blue line* represents an investor less risky than the counterparty. All values are in basis points.

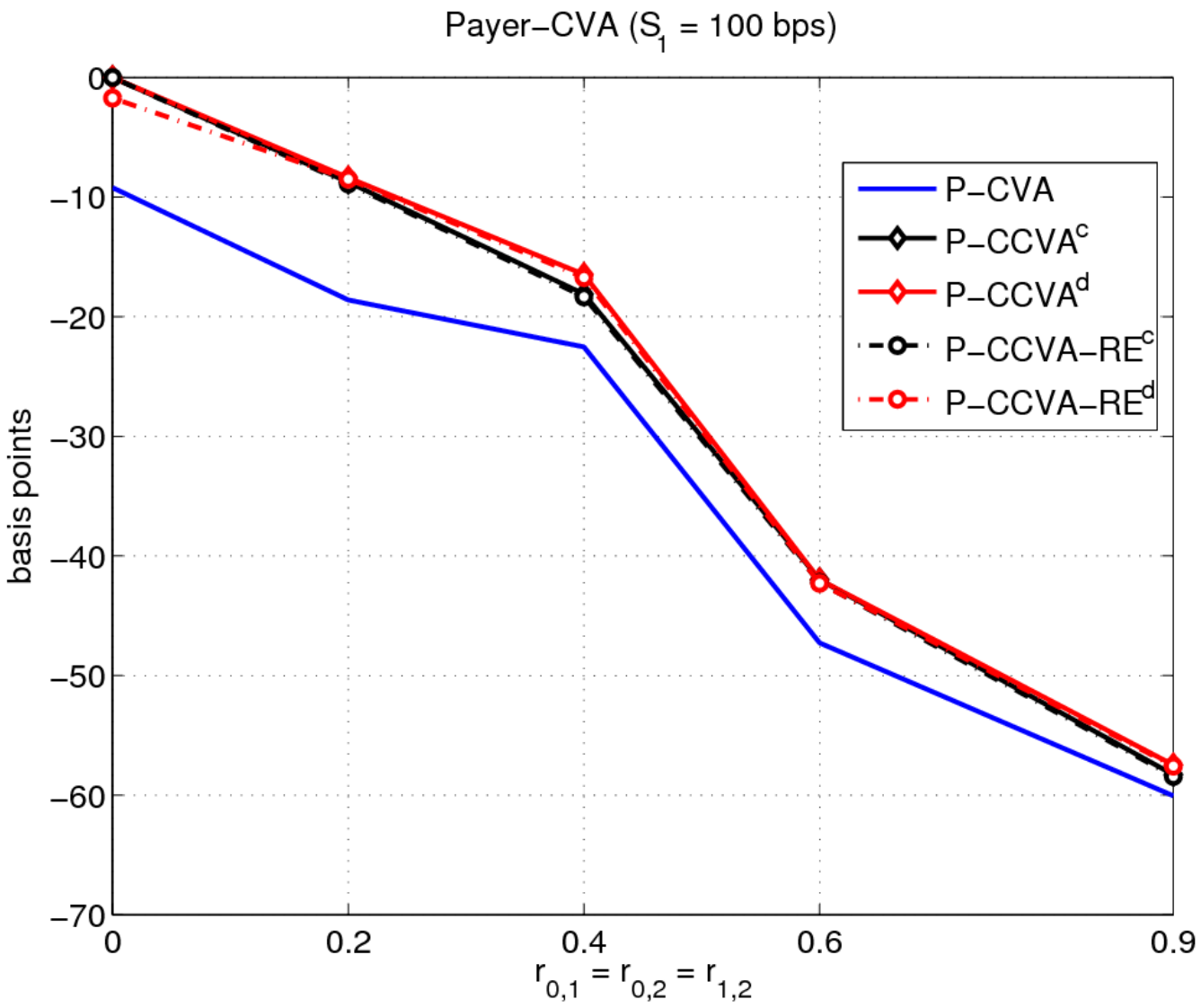
See the full paper by Brigo, Capponi, Pallavicini and Papatheodorou ‘Collateral Margining in Arbitrage-Free Counterparty Valuation Adjustment including Re-Hypotecation and Netting”
available at `http://arxiv.org/abs/1101.3926`

Figure explanation

From the fig, we see that the case of an investor riskier than the counterparty (M/H) leads to positive value for DVA-CVA, while the case of an investor less risky than the counterparty has the opposite behaviour. If we inspect the DVA and CVA terms as in the paper we see that when the investor is riskier the DVA part of the correction dominates, while when the investor is less risky the counterparty has the opposite behaviour.
Re-hypothecation enhances the absolute size of the correction, a reasonable behaviour, since, in such case, each party has a greater risk because of being unsecured on the collateral amount posted to the other party in case of default.

Let us now look at a case with more contagion: a CDS.

Collate


Payer−CVA (S_1 = 100 bps)
basis points
0
−10
−20
−30
−40
−50
−60
−70
0
0.2
0.4
0.6
0.9
$r_{0,1} = r_{0,2} = r_{1,2}$
P−CVA
P−CCVA^c
P−CCVA^d
P−CCVA−RE^c
P−CCVA−RE^d

Collateral Management and Gap Risk II

The figure refers to a payer CDS contract as underlying. See the full paper by Brigo, Capponi and Pallavicini (2011) for more cases.

If the investor holds a payer CDS, he is buying protection from the counterparty, i.e. he is a protection buyer.

We assume that the spread in the fixed leg of the CDS is 100 while the initial equilibrium spread is about 250.

Given that the payer CDS will be positive in most scenarios, when the investor defaults it is quite unlikely that the net present value be in favor of the counterparty.

We then expect the CVA term to be relevant, given that the related option will be mostly in the money. This is confirmed by our outputs.

Collateral Management and Gap Risk III

We see in the figure a relevant CVA component (part of the bilateral DVA - CVA) starting at 10 and ending up at 60 bps when under high correlation.

We also see that, for zero correlation, collateralization succeeds in completely removing CVA, which goes from 10 to 0 basis points.

However, collateralization seems to become less effective as default dependence grows, in that collateralized and uncollateralized CVA become closer and closer, and for high correlations we still get 60 basis points of CVA, even under collateralization.

The reason for this is the instantaneous default contagion that, under positive dependency, pushes up the intensity of the survived entities, as soon as there is a default of the counterparty.

Collateral Management and Gap Risk IV

Indeed, the term structure of the on-default survival probabilities (see paper) lies significantly below the one of the pre-default survival probabilities conditioned on $\mathcal{G}_{\tau-}$, especially for large default correlation.

The result is that the default leg of the CDS will increase in value due to contagion, and instantaneously the Payer CDS will be worth more. This will instantly increase the loss to the investor, and most of the CVA value will come from this jump.

Given the instantaneous nature of the jump, the value at default will be quite different from the value at the last date of collateral posting, before the jump, and this explains the limited effectiveness of collateral under significantly positive default dependence.

Monitoring Counterparty Credit Risk

- When we monitor a (symmetric) risk in a bilateral agreement, we should introduce a "metric" which is shared by both parties.
  - → The ISDA Master Agreement defines the term *exposure* to be the netted mid-market mark-to-market value of the transaction.
- We name the exposure priced at time t, either by the investor or by the counterparty, with ε_t.
- Notice that the ISDA Master Agreement allows the calculation agent to be a third party.
- Since counterparty risk can be sized in term of exposure, we can operate to mitigate the risk by reducing such quantity.

Mitigating Counterparty Credit Risk – I

- The ISDA Master Agreement lists two different tools to reduce exposure:
  - → close-out netting rules, which state that if a default occurs, multiple obligations between two parties are consolidated into a single net obligation; and
  - → collateralization, namely the right of recourse to some asset of value that can be sold or the value of which can be applied in the event of default on the transaction.
- We consider that assets used as collaterals are posted on a Collateral Account held by a Collateral Taker, and we name its value at time t with C_t.
- Notice that if at time t the investor posts some collateral we consider that $dC_t < 0$, the other way round if the counterparty is posting.

Mitigating Counterparty Credit Risk – II

- In the following we assume that close-out netting rules are always active, so that we consider the transaction $\Pi(t, T)$ and the collateral account C_t together when calculating the CVA.
- Thus, under close-out netting rules we get

$$\text{CVA}(t, T; C) := \mathbb{E}_t\big[\,\bar{\Pi}(t, T; C) - \Pi(t, T) - C_T D(t, T)\,\big]$$

  where the expectation is taken under risk-neutral measure, and $\bar{\Pi}(t, T; C)$ will be analyzed in the following slides.
- Furthermore, we assume that mid-market exposure ε_t can be calculated from the risk-free $\Pi(t, T)$ as

$$\varepsilon_t \doteq \mathbb{E}_t[\,\Pi(t, T)\,]$$

Re-hypothecation Liquidity Risk – I

- At transaction maturity or after applying close-out netting, the originating party expects to get back the remaining collateral.
- Yet, prevailing legislations may give to the Collateral Taker some rights on the collateral itself.
- For instance, on an early termination date a counterparty to an English CSA will find itself as an unsecured creditor, thus entitled to only a fraction of the value of the collateral it transferred.
- With a New York CSA transferred cash collateral or re-hypothecated collateral are both likely to leave the collateral provider in the same position as an unsecured creditor, but, in this case, the parties may agree on amending the provisions of the CSA which make re-hypothecation possible.

Re-hypothecation Liquidity Risk – II

- In case of collateral re-hypothecation the surviving party must consider the possibility to recover only a fraction of his collateral.
  - → We name such recovery rate Rec'_I, if the investor is the Collateral Taker, or Rec'_C in the other case (we often use $\text{Lgd}'_I := 1 - \text{Rec}'_I$ and $\text{Lgd}'_C := 1 - \text{Rec}'_C$).
- In the worst case the surviving party has no precedence on other creditors to get back his collateral. In such case the recovery rate of collateral is the one of the transaction. Thus, we get

$$\text{Rec}_I \leq \text{Rec}'_I \leq 1 \ , \quad \text{Rec}_C \leq \text{Rec}'_C \leq 1$$

- If the Collateral Taker is a risk-free third-party we can assume that $\text{Rec}'_I = \text{Rec}'_C = 1$.

Collateral Choice

Ideally, firms would like an asset of stable and predictable value, an asset that is not linked to the value of the transaction in any way and an asset that can be sold quickly and easily if the need arises. [ISDA, Coll. Review, 1.1]

- Thus, in order to achieve an effective collateralization of the transaction, we require that
  - → collaterals hedge investor's exposure on counterparty's default event,
  - → they are liquid assets,
  - → they are not related to the deal's underlying assets or to the counterparty.
- In practice, when collaterals do not match such requirements, their value is reduced by means of corrective factors named haircuts.

Margining Practice – I

- In general, margining practice consists in a pre-fixed set of dates during the life of a deal when both parties post or withdraw collaterals, according to their current exposure, to or from an account held by the Collateral Taker.
- The Collateral Taker may be a third party or the party of the transaction who is not posting collateral.
- Notice that in legal documents where a pledge or a security interest is in act the Collateral Taker is named the Secured Party, while the other party is the Pledgor.
- We do not consider legal issues which may change collateral arrangement (pledge vs. title transfer) but for re-hypothecation issues.

Margining Practice – II

- The Collateral Taker remunerates the account (usually) at over-night rate.
  - $\rightarrow$ In the following we consider that the collaterals are risk-free and their account is a cash account accruing at risk-free rate.
- At deal termination date the parties are not forced to close the collateral account, but they may agree to use it for a new deal.
  - $\rightarrow$ We consider that the collateral account is opened anew for each new deal and it is closed upon a default event occurs or maturity is reached.
- If the account is closed any collateral held by the Collateral Taker would be required to be returned to the originating party.
  - $\rightarrow$ We have $C_u = 0$ for all $u \leq t$ or $u \geq T$.
- We do not consider haircuts in the following.

Margining Practice – III

- A realistic margining practice should allow for collateral posting only on a fixed time-grid ($t_0 = t, \ldots, t_N = T$), and for the presence of independent amounts (A), minimum transfer amounts (M), and thresholds (H), with $H \geq M$.
- Independent amounts represent a further insurance on the transaction and they are often posted as an upfront protection, but they may be updated according to exposure changes. We do not consider them in the following.
- Thresholds represent the amount of permitted unsecured risk, so that they may depend on the credit quality of the counterparties.
- Moving thresholds depending on a deterioration of the credit quality of the counterparties (downgrade triggers) have been a source of liquidity strain during the market crisis.

Margining Practice – IV

- At each collateral posting date t_i, the collateral account is updated according to changes in exposure, otherwise producing an unsecured risk.

1. First, we consider how much collateral the investor should post to or withdraw from the collateral account:

$$1_{\{|(\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-}|>M\}}((\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-})$$

2. Then, we consider how much collateral the counterparty should post to or withdraw from the collateral account:

$$1_{\{|(\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-}|>M\}}((\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-})$$

Margining Practice – V

- By adding the two terms we get how the collateral account is updated during the life of the transaction

$$C_{t_0} := 0\,, \quad C_{t_N^+} := 0\,, \quad C_{u^-} := \frac{C_{\beta(u)^+}}{D(\beta(u), u)}$$

$$\begin{aligned} C_{t_i^+} := C_{t_i^-} & \\ & + 1_{\{|(\varepsilon_{t_i}+H_I)^- - C^-_{t_i^-}|>M\}}((\varepsilon_{t_i} + H_I)^- - C^-_{t_i^-}) \\ & + 1_{\{|(\varepsilon_{t_i}-H_C)^+ - C^+_{t_i^-}|>M\}}((\varepsilon_{t_i} - H_C)^+ - C^+_{t_i^-}) \end{aligned}$$

where $\beta(u)$ is the last update time before u, and $t_0 < u \leq t_N$.

- In case of no thresholds ($H_I = H_C = 0$) and no minimum transfer amount ($M = 0$), we obtain a simpler rule

$$C_{t_0} = C_{t_N^+} = 0\,, \quad C_{t^-} = \frac{\varepsilon_{\beta(u)}}{D(\beta(u), u)}\,, \quad C_{t_i^+} = \varepsilon_{t_i}$$

Close-Out Netting Rules – I

The effect of close-out netting is to provide for a single net payment requirement in respect of all the transactions that are being terminated, rather than multiple payments between the parties. Under the applicable accounting rules and capital requirements of many jurisdictions, the availability of close-out netting allows parties to an ISDA Master Agreement to account for transactions thereunder on a net basis. [ISDA, Coll. Review, 2.1.1]

- The occurrence of an event of default gives the parties the right to terminate all transactions that are concluded under the relevant ISDA Master Agreement.
- The ISDA Master Agreement provides for the mechanism of close-out netting to be enforced.

Close-Out Netting Rules – II

The Secured Party will transfer to the Pledgor any proceeds and posted credit support remaining after liquidation and/or set-off after satisfaction in full of all amounts payable by the Pledgor with respect to any obligations; the Pledgor in all events will remain liable for any amounts remaining unpaid after any liquidation and/or set-off. [ISDA, CSA Annex, 8]

- In case of default of one party, the surviving party should evaluate the transactions just terminated, due to the default event occurrence, to claim for a reimbursement after the application of netting rules to consolidate the transactions, inclusive of collateral accounts.
  - → The ISDA Master Agreement defines the term *close-out amount* to be the amount of the losses or costs of the surviving party would incur in replacing or in providing for an economic equivalent.

Close-Out Netting Rules – III

- Notice that the close-out amount is not a symmetric quantity w.r.t. the exchange of the role of two parties, since it is valued by one party after the default of the other one.
- Instead of the close-out amount we introduce the ”on-default exposure”, namely the price of the replacing transaction or of its economic equivalent.
- We name the on-default exposure priced at time t by the investor on counterparty's default with $\varepsilon_{I,t}$ (and $\varepsilon_{C,t}$ in the other case, namely when the investor is defaulting). Notice that we always consider all prices from the point of view of the investor. Thus,
  - $\longrightarrow$ a positive value for $\varepsilon_{I,t}$ means the investor is a creditor of the counterparty, while
  - $\longrightarrow$ a negative value for $\varepsilon_{C,t}$ means the counterparty is a creditor of the investor.

Cash Flows on Counterparty Default Event – I

- We start by listing all the situations may arise on counterparty default event. The case of the investor's default event will be derived accordingly.
- Our goal is to calculate the present value of all cash flows involved by the contract by taking into account:
  - $\longrightarrow$ collateral margining operations, and
  - $\longrightarrow$ close-out netting rules in case of default.
- Notice that we can safely aggregate the cash flows of the contract with the ones of the collateral account, since on contract termination all the posted collateral are returned to the originating party.
- We introduce the (first) default time $\tau := \min\{\tau_C, \tau_I\}$.

Cash Flows on Counterparty Default Event – II

1. The investor measures a positive (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} > 0$), and some collateral posted by the counterparty is available ($C_{\tau_C} > 0$).
   - $\rightarrow$ Then, the exposure is reduced by netting, and the remaining collateral (if any) is returned to the counterparty. If the collateral is not enough, the investor suffers a loss for the remaining exposure.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}>0\}} 1_{\{C_\tau>0\}} (\mathrm{Rec}_C(\varepsilon_{I,\tau} - C_\tau)^+ + (\varepsilon_{I,\tau} - C_\tau)^-)$$

Cash Flows on Counterparty Default Event – III

2. The investor measures a positive (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} > 0$), and some collateral posted by the investor is available ($C_{\tau_C} < 0$).
   - → Then, the investor suffers a loss for the whole exposure. All the collateral (if any) is returned to the investor if it is not re-hypothecated, otherwise an unsecured claim is needed.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}>0\}} 1_{\{C_\tau<0\}} (\mathrm{Rec}_C \varepsilon_{I,\tau} - \mathrm{Rec}'_C C_\tau)$$

Cash Flows on Counterparty Default Event – IV

3. The investor measures a negative (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} < 0$), and some collateral posted by the counterparty is available ($C_{\tau_C} > 0$).
   - $\longrightarrow$ Then, the exposure is paid to the counterparty, and the counterparty gets back its collateral in full.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}<0\}} 1_{\{C_\tau>0\}} (\varepsilon_{I,\tau} - C_\tau)$$

Cash Flows on Counterparty Default Event – V

4. The investor measures a negative (on-default) exposure on counterparty default ($\varepsilon_{I,\tau_C} < 0$), and some collateral posted by the investor is available ($C_{\tau_C} < 0$).
   - $\longrightarrow$ Then, the exposure is reduced by netting and paid to the counterparty. The investor gets back its remaining collateral (if any) in full if it is not re-hypothecated, otherwise an unsecured claim is needed for the part of collateral exceeding the exposure.

$$1_{\{\tau=\tau_C<T\}} 1_{\{\varepsilon_{I,\tau}<0\}} 1_{\{C_\tau<0\}} \left((\varepsilon_{I,\tau} - C_\tau)^- + \mathrm{Rec}'_C (\varepsilon_{I,\tau} - C_\tau)^+ \right)$$

Aggregating Cash Flows – I

- Now, we can aggregate all these cash flows, along with cash flows coming from the default of the investor and the ones due in case of non-default, inclusive of the cash-flows of the collateral account.
- We obtain the cash flows coming from the default of the investor simply by reformulating the previous line of reasoning from the point of view of the counterparty.
- In the following equations we use the risk-free discount factor $D(t, T)$, which is implicitly used also in the definitions of the risk-free discounted payoff $\Pi(t, T)$, and in the accumulation curve used for the collateral account C_t.

Aggregating Cash Flows – II

- We obtain by summing all the contributions

$$
\begin{aligned}
\bar{\Pi}(t,T;C) = & \\
& 1_{\{\tau>T\}}\Pi(t,T) \\
& + 1_{\{\tau<T\}}(\Pi(t,\tau)+D(t,\tau)C_\tau) \\
& + 1_{\{\tau=\tau_C<T\}}D(t,\tau)1_{\{\varepsilon_{I,\tau}<0\}}1_{\{C_\tau>0\}}(\varepsilon_{I,\tau}-C_\tau) \\
& + 1_{\{\tau=\tau_C<T\}}D(t,\tau)1_{\{\varepsilon_{I,\tau}<0\}}1_{\{C_\tau<0\}}((\varepsilon_{I,\tau}-C_\tau)^- + \mathrm{Rec}'_C(\varepsilon_{I,\tau}-C_\tau)^+) \\
& + 1_{\{\tau=\tau_C<T\}}D(t,\tau)1_{\{\varepsilon_{I,\tau}>0\}}1_{\{C_\tau>0\}}((\varepsilon_{I,\tau}-C_\tau)^- + \mathrm{Rec}_C(\varepsilon_{I,\tau}-C_\tau)^+) \\
& + 1_{\{\tau=\tau_C<T\}}D(t,\tau)1_{\{\varepsilon_{I,\tau}>0\}}1_{\{C_\tau<0\}}(\mathrm{Rec}_C\varepsilon_{I,\tau} - \mathrm{Rec}'_C C_\tau) \\
& + 1_{\{\tau=\tau_I<T\}}D(t,\tau)1_{\{\varepsilon_{C,\tau}>0\}}1_{\{C_\tau<0\}}(\varepsilon_{C,\tau}-C_\tau) \\
& + 1_{\{\tau=\tau_I<T\}}D(t,\tau)1_{\{\varepsilon_{C,\tau}>0\}}1_{\{C_\tau>0\}}((\varepsilon_{C,\tau}-C_\tau)^+ + \mathrm{Rec}'_I(\varepsilon_{C,\tau}-C_\tau)^-) \\
& + 1_{\{\tau=\tau_I<T\}}D(t,\tau)1_{\{\varepsilon_{C,\tau}<0\}}1_{\{C_\tau<0\}}((\varepsilon_{C,\tau}-C_\tau)^+ + \mathrm{Rec}_I(\varepsilon_{C,\tau}-C_\tau)^-) \\
& + 1_{\{\tau=\tau_I<T\}}D(t,\tau)1_{\{\varepsilon_{C,\tau}<0\}}1_{\{C_\tau>0\}}(\mathrm{Rec}_I\varepsilon_{C,\tau} - \mathrm{Rec}'_I C_\tau)
\end{aligned}
$$

Aggregating Cash Flows – III

- Hence, by a straightforward calculation we get

$$
\begin{aligned}
\bar{\Pi}(t,T;C) = \Pi(t,T) & \\
& - 1_{\{\tau<T\}} D(t,\tau)\left(\Pi(\tau,T) - 1_{\{\tau=\tau_C\}}\varepsilon_{I,\tau} - 1_{\{\tau=\tau_I\}}\varepsilon_{C,\tau}\right) \\
& - 1_{\{\tau=\tau_C<T\}} D(t,\tau)(1-\mathrm{Rec}_C)(\varepsilon^+_{I,\tau} - C^+_\tau)^+ \\
& - 1_{\{\tau=\tau_C<T\}} D(t,\tau)(1-\mathrm{Rec}'_C)(\varepsilon^-_{I,\tau} - C^-_\tau)^+ \\
& - 1_{\{\tau=\tau_I<T\}} D(t,\tau)(1-\mathrm{Rec}_I)(\varepsilon^-_{C,\tau} - C^-_\tau)^- \\
& - 1_{\{\tau=\tau_I<T\}} D(t,\tau)(1-\mathrm{Rec}'_I)(\varepsilon^+_{C,\tau} - C^+_\tau)^-
\end{aligned}
$$

- Notice that the collateral account enters only as a term reducing the exposure of each party upon default of the other one, keeping into account which is the party who posted the collateral.

Collateralized Bilateral CVA

- Now, by taking risk-neutral expectation of both sides of the above equation, and by plugging in the definition of mid-market exposure, we obtain the general expression for collateralized bilateral CVA.

$$
\begin{aligned}
\mathrm{CVA}(t,T;C) = & -\mathbb{E}_t\Big[\, 1_{\{\tau<T\}} D(t,\tau)\left(\varepsilon_\tau - 1_{\{\tau=\tau_C\}}\varepsilon_{I,\tau} - 1_{\{\tau=\tau_I\}}\varepsilon_{C,\tau}\right)\Big] \\
& - \mathbb{E}_t\Big[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}_C(\varepsilon^+_{I,\tau} - C^+_\tau)^+ \Big] \\
& - \mathbb{E}_t\Big[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}'_C(\varepsilon^-_{I,\tau} - C^-_\tau)^+ \Big] \\
& - \mathbb{E}_t\Big[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}_I(\varepsilon^-_{C,\tau} - C^-_\tau)^- \Big] \\
& - \mathbb{E}_t\Big[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}'_I(\varepsilon^+_{C,\tau} - C^+_\tau)^- \Big]
\end{aligned}
$$

- Now, we need a recipe to calculate on-default exposures ε_{I,τ_C} and ε_{C,τ_I}, that, in the practice, are approximated from today exposure corrected for haircuts or add-ons.

Formulae for Collateralized Bilateral CVA – I

- We consider all the exposures being evaluated at mid-market, namely we consider:

$$\varepsilon_{I,t} \doteq \varepsilon_{C,t} \doteq \varepsilon_t$$

- Thus, in such case we obtain for collateralized bilateral CVA

$$\begin{aligned}
\text{CVA}(t,T;C) = & -\mathbb{E}_t\left[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\text{LGD}_C(\varepsilon_\tau^+ - C_\tau^+)^+ \,\right] \\
& - \mathbb{E}_t\left[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\text{LGD}'_C(\varepsilon_\tau^- - C_\tau^-)^+ \,\right] \\
& - \mathbb{E}_t\left[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\text{LGD}_I(\varepsilon_\tau^- - C_\tau^-)^- \,\right] \\
& - \mathbb{E}_t\left[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\text{LGD}'_I(\varepsilon_\tau^+ - C_\tau^+)^- \,\right]
\end{aligned}$$

- After this section we show a possible way to relax such approximation.

Formulae for Collateralized Bilateral CVA – II

- If collateral re-hypothecation is not allowed ($\text{LGD}'_C \doteq \text{LGD}'_I \doteq 0$) the above formula simplifies to

$$\begin{aligned}\text{CVA}(t,T;C) = &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_C<T\}}D(t,\tau)\text{LGD}_C(\varepsilon_\tau^+ - C_\tau^+)^+\,\right]\\ &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_I<T\}}D(t,\tau)\text{LGD}_I(\varepsilon_\tau^- - C_\tau^-)^-\,\right]\end{aligned} \tag{3}$$

- On the other hand, if re-hypothecation is allowed and the surviving party always faces the worst case ($\text{LGD}'_C \doteq \text{LGD}_C$ and $\text{LGD}'_I \doteq \text{LGD}_I$), we get

$$\begin{aligned}\text{CVA}(t,T;C) = &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_C<T\}}D(t,\tau)\text{LGD}_C(\varepsilon_\tau - C_\tau)^+\,\right]\\ &-\mathbb{E}_t\left[\,1_{\{\tau=\tau_I<T\}}D(t,\tau)\text{LGD}_I(\varepsilon_\tau - C_\tau)^-\,\right]\end{aligned} \tag{4}$$

Formulae for Collateralized Bilateral CVA – III

- If we remove collateralization ($C_t = 0$), we recover the result of Brigo and Capponi (2008), and used in Brigo, Pallavicini and Papatheodorou (2009).

$$\begin{aligned} \mathrm{CVA}^{\mathrm{BC}}(t,T) = & -\mathbb{E}_t\left[\, 1_{\{\tau=\tau_C<T\}} D(t,\tau)\mathrm{LGD}_C \varepsilon_\tau^+ \,\right] \\ & - \mathbb{E}_t\left[\, 1_{\{\tau=\tau_I<T\}} D(t,\tau)\mathrm{LGD}_I \varepsilon_\tau^- \,\right] \end{aligned} \tag{5}$$

- If we remove collateralization ($C_t = 0$) and we consider a risk-free investor ($\tau_I \to \infty$), we recover the result of Brigo and Pallavicini (2007), but see also Canabarro and Duffie (2004).

$$\mathrm{CVA}^{\mathrm{BP}}(t,T) = -\mathbb{E}_t\left[\, 1_{\{\tau_C<T\}} D(t,\tau_C)\mathrm{LGD}_C \varepsilon_{\tau_C}^+ \,\right] \tag{6}$$

An Example: Perfect Collateralization

- We consider, for this example, updating the collateral account continuously. We obtain the following (perfect) collateralization rule.

$$C_t^{\text{perfect}} := \varepsilon_t$$

- Thus, if we plug it into the collateralized bilateral CVA equation (with all exposure at mid-market), we get that all terms drop, as expected, leading to

$$\text{CVA}(t, T; C^{\text{perfect}}) = 0$$

$$\mathbb{E}_t\big[\,\bar{\Pi}(t, T; C)\,\big] = \mathbb{E}_t[\,\Pi(t, T)\,] = \varepsilon_t = C_t^{\text{perfect}}$$

- Thus, the proper discount curve for pricing the deal is the collateral accrual curve (see also Fujii et al. (2010) or Piterbarg (2010)).

Basel III and CVA I

"Under Basel II, the risk of counterparty default and credit migration risk were addressed but mark-to-market losses due to credit valuation adjustments (CVA) were not. During the financial crisis, however, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults."

Basel Committee on Banking Supervision, BIS (2011). Press release available at *http://www.bis.org/press/p110601.pdf*

Given the above situation, Basel III is imposing very severe capital requirements for CVA.

This may lead to forms of securitization of CVA such as margin lending on the whole exposure or on tranches of the exposure.

Basel III and CVA II

Such ”securitization of CVA” would be very difficult to model and to manage, requiring a global valuation perspective.

Few papers have appeared in the literature that are attempting to address CVA securitization, see for example Albanese et al (2011).

The problem with the traditional upfront charge for unilateral CVA is that it leaves CVA volatility with the investor and not with the risky counterparty.

In the unilateral case, the investor charges an upfront for CVA to the counterparty and then implements a hedging strategy. The investor is thus exposed to CVA mark to market volatility in the future.

Basel III and CVA III

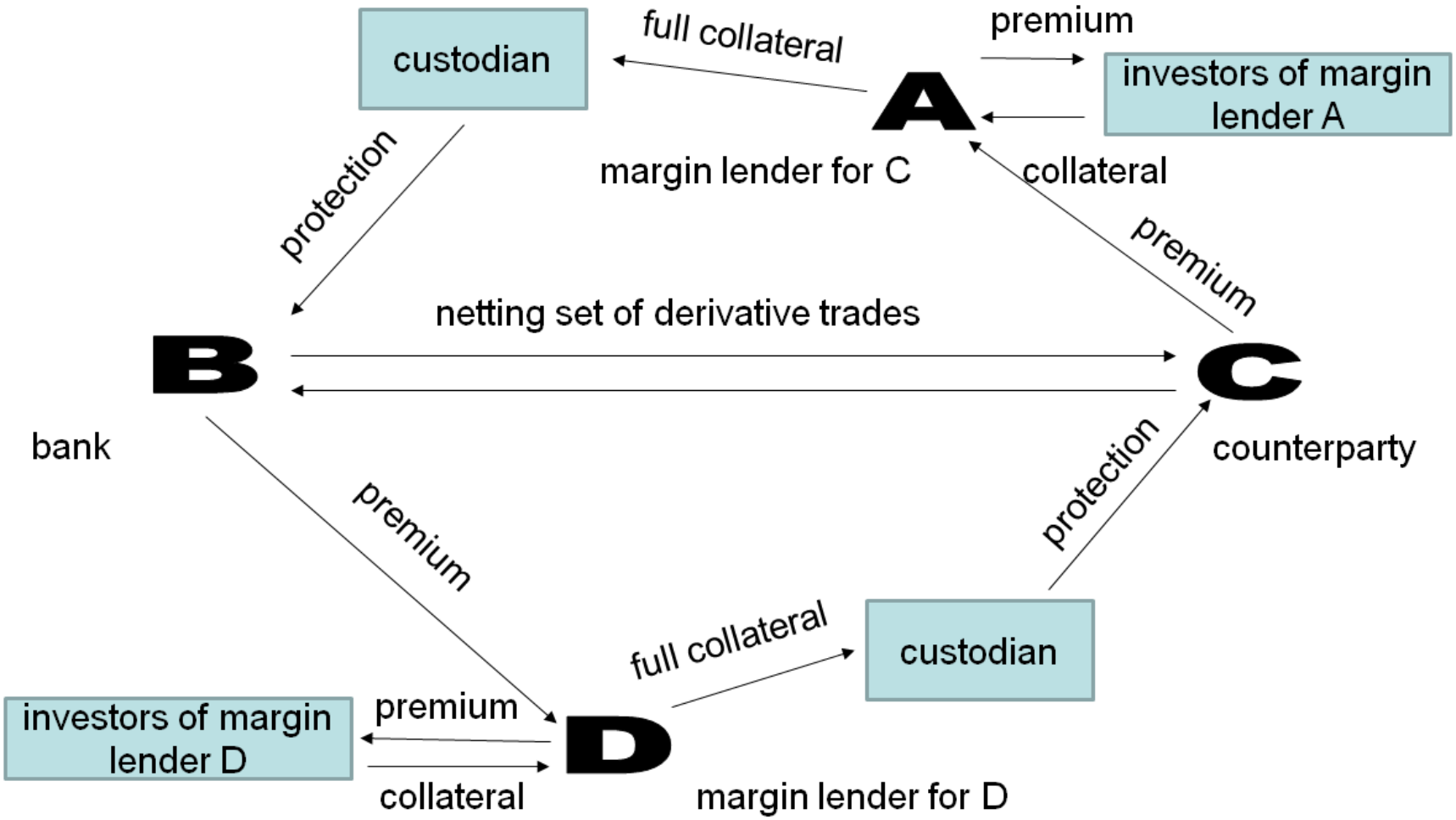
Alternatively the investor may request collateral from the counterparty, but not all counterparties are able to regularly post collateral, and this can be rather punitive for some corporate counterparties.

See recent example on Lufthansa from Risk magazine:

> *The airline's Cologne-based head of finance, Roland Kern, expects its earnings to become more volatile not because of unpredictable passenger numbers, interest rates or jet fuel prices, but because it does not post collateral in its derivatives transactions".*

Margin lending is a possible solution this problem.

Margin Lending I



Margin Lending II

Traditionally, the CVA is typically charged by the structuring bank *B* (investor) either on an upfront basis or it is built into the structure as a fixed coupon stream.

Margin lending instead is predicated on the notion of floating rate CVA.

Assume we are in a bi-partite transaction between the default-free bank/investor B and the defaultable counterparty (say a corporate client) C. The bank may require a CVA payment at time 0 for protection on the exposure up to 6 months. Then in 6 months the bank will require a CVA payment for protection for further six months, prevailing at that time, on what will be the exposure up to one year, and on and on, up to the final maturity.

Such a CVA would be a Floating rate CVA.

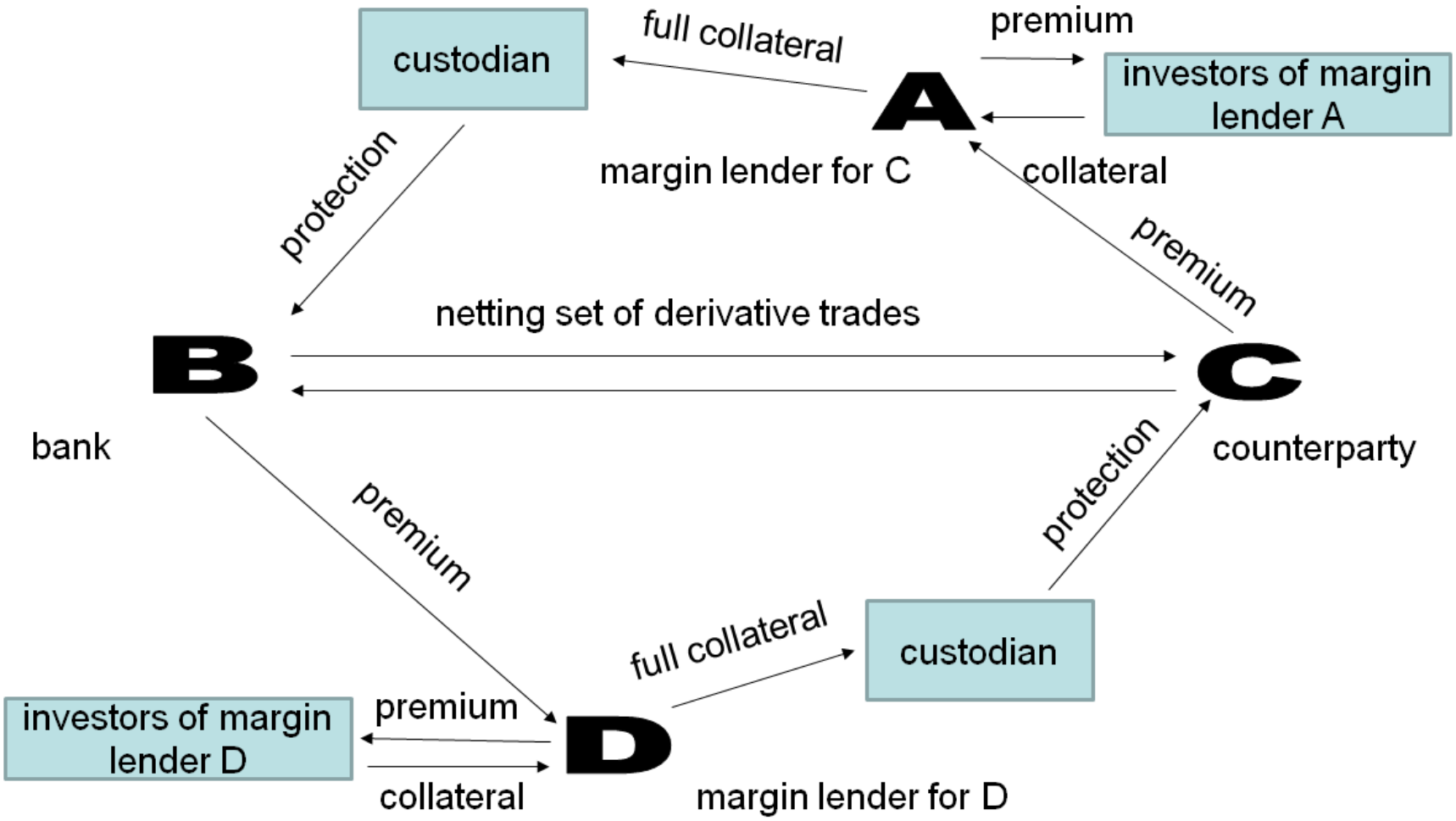
Margin Lending III

Margin lending is designed in such a way to transfer the conditional credit spread volatility risk and the mark-to-market volatility risk, or in other terms CVA volatility, from the bank to the counterparties.

Margin Lending IV

We may explain this more in detail by following the arrows in the Figure.

Margin Lending V



Margin Lending VI

To avoid posting collateral, C enters into a margin lending transaction. C pays periodically (say semi-annually) a floating rate CVA to margin lender A ('premium' arrow connecting C to A), which A pays to investors (premium arrow connecting A to Investors). This latest payment can have a seniority structure similar to that of a cash CDO.

In exchange, for six months the investors provide A with daily collateral posting ('collateral' arrow connecting Investors to A) and A passes the collateral to a custodian ('collateral' arrow connecting A to the custodian).

This collateral need not be cash, but it can be in the form of hypothecs

If C defaults within the semi-annual period, the collateral is paid to B to provide protection ('protection' arrow connecting the custodian to B) and the loss in taken by the Investors who provided the collateral.

Margin Lending VII

At the end of the six months period, the margin lender may decide whether to continue with the deal or to back off. With this mechanism C is bearing the CVA volatility risk, whereas B is not exposed to CVA volatility risk, which is the opposite of what happens with traditional upfront CVA charges.

Albanese, Brigo and Oertel (2011) argue that whenever an entity's credit worsens, it receives a subsidy from its counterparties in the form of a DVA positive mark to market which can be monetized by the entity's bond holders only upon their own default.

Margin Lending VIII

Whenever an entity's credit improves instead, it is effectively taxed as its DVA depreciates.

Wealth is thus transferred from the equity holders of successful companies to the bond holders of failing ones, the transfer being mediated by banks acting as financial intermediaries and implementing the traditional CVA/DVA mechanics.

Rewarding failing firms with a cash subsidy may be a practice of debatable merit as it skews competition. But rewarding failing firms with a DVA benefit is without question suboptimal from an economic standpoint: the DVA benefit they receive is paid in cash from their counterparties but, once received in this form, it cannot be invested and can only be monetized by bond holders upon default.

Margin Lending IX

Again, Albanese, Brigo and Oertel (2011) submit that margin lending structures may help reversing the macroeconomic effect by eliminating long term counterparty credit risk insurance and avoiding the wealth transfer that benefits the bond holders of defaulted entities.

There are a number of possible problems with this. First, proper valuation and hedging of this to the investor who are providing collateral to the lender is going to be tough. There is no satisfactory standard for even simple synthetic CDOs.

Admittedly this requires and effective global valuation framework, see for example the discussion in Albanese et al (2011).

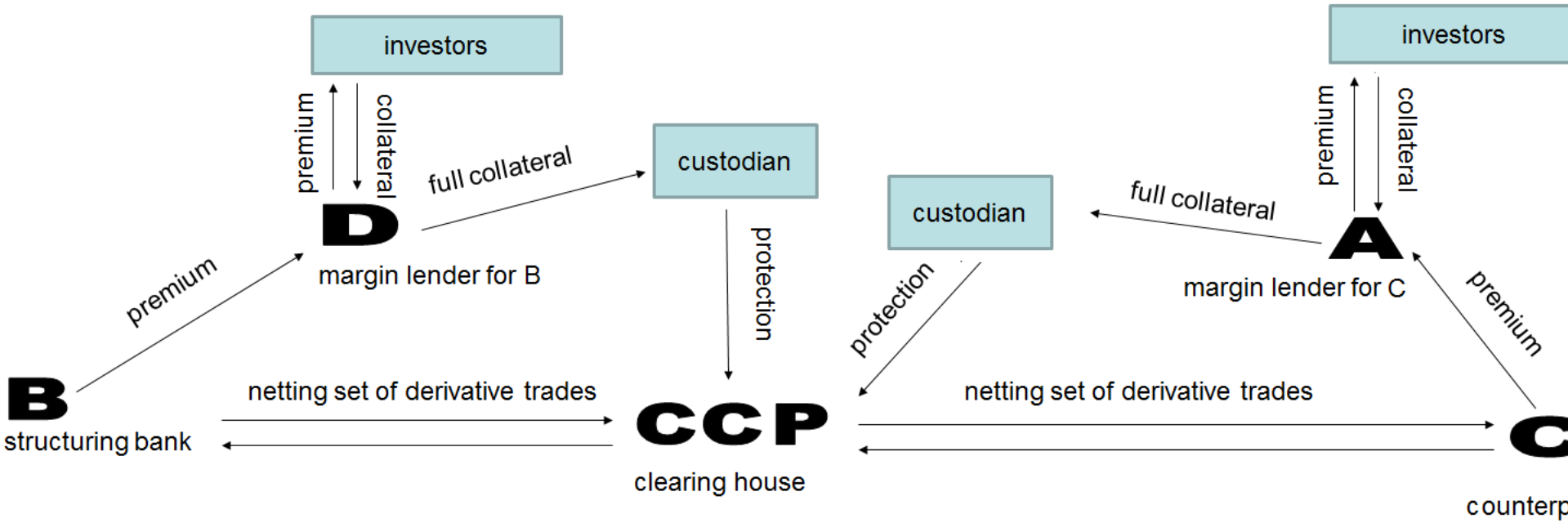
The other problem is: what if all margin lenders pull off at some point due to a systemic crisis?

Margin Lending X

That would be a problem, indeed, but one may argue that the market is less likely to arrive in such a situation in the first place if the wrong incentives to defaulting firms are stopped and an opposite structure, such as the one in margin lending, is implemented.

There is also a penta-partite version including a clearing house. But there is much more work to do to assess this framework properly.

Margin Lending XI



CVA Restructuring: Global Valuation? I

A fair valuation and risk management of CVA restructuring through margin lending requires a global model, in order to have consistency and sensible greeks

But even when staying with traditional uprfont CVA and DVA in large portfolios, as our examples above pointed out, different models are typically used in different asset classes. This can lead to models that are inconsistent with each other.

For example, our equity example above used a firm value model, whereas in the other asset classes we used reduced form models.

CVA Restructuring: Global Valuation? II

What if one has a portfolio with all asset classes together? And more generally, how does one ensure a consistent modeling framework that is needed to get meaningful prices and especially cross correlation sensitivities?

The problem is rather difficult and involves important computational resources and intelligent systems architecture.

Few papers have appeared in the literature that are attempting a global valuation framework, see for example Albanese et al (2010, 2011).

Delicate points include:

Modeling dependencies across defaults (we do not have even a good model for synthetic corporate CDO, base correlation still used there, see for example Brigo Pallavicini and Torresetti (2010))

CVA Restructuring: Global Valuation? III

Modeling dependencies between defaults and each other asset class

Modeling dependencies between different asset classes

Properly including credit volatility with positive credit spreads

Conclusions I

- Counterparty Risk adds one level of optionality.
- Analysis including underlying asset/ counterparty default correlation requires a credit model.
- Highly specialized hybrid modeling framework.
- Accurate scenarios for wrong way risk.
- Outputs vary and can be very different from Basel II multipliers
- Outputs are strongly model dependent and involve model risk and model choices
- Bilateral CVA brings in symmetry but also paradoxical statements
- Bilateral CVA requires a choice of closeout (risk free or substitution), and this is relevant.
- The DVA term in bilateral CVA is hard to hedge, especially in the jump-to-default risk component.

Conclusions II

- Approximations ignoring first to default risk (sometimes used in the industry) do not work well.
- Inclusion of Collateral and netting rules is possible
- Gap risk in collateralization remains relevant in presence of strong contagion
- Basel III will make capital requirements rather severe
- Contingent CDS as hedging instruments have limited effectiveness
- CVA restructuring through margin lending and hypothecs is a possible alternative
- Proper valuation and management of CVA and especially CVA restructuring requires a Consistent Global Valuation approach
- This also holds for possible forms of CVA Securitization

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