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Geometry of polar wedges in Riesz spaces and super-replication prices in incomplete financial markets. (English summary)

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In a general semimartingale model for incomplete financial markets, F. Delbaen and W. Schachermayer [Math. Ann. **312** (1998), no. 2, 215–250; [MR1671792 \(99k:60117\)](#)] proved that, under the condition of “No Free Lunch with Vanishing Risk”, for claims f which are bounded from below, the classical super-replication price is equal to $\sup_{Q \in \mathcal{M}} E_Q[f]$, \mathcal{M} being the set of all pricing measures. However this duality breaks down for claims which are unbounded from below, which is why, e.g., S. Biagini and M. Frittelli [Ann. Appl. Probab. **14** (2004), no. 4, 1970–1991; [MR2099659 \(2005h:91096\)](#)] suggested a less expensive “weak super-replication price” and characterized the subset of pricing measures which allow the dual representation.

In this paper, by considering the pure geometric structure of the underlying problem (based on the properties of Riesz spaces), the authors generalize the main results given by Biagini and Frittelli [op. cit.]. In particular, they extend the definition of super-replication price, considering arbitrary convex cones of terminal wealths in place of the attainable wealths arising from admissible trading strategies, and provide dual representation results by means of suitable sets of separating measures.

Reviewed by [Beatrice Acciaio](#)

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