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Let α be a tensor norm, X, Y be Banach spaces, and $z \in X \otimes Y$. Then the finite hull $\overrightarrow{\alpha}(z; X, Y) = \inf\{\alpha(z; E, F) \colon E, F$ are finite-dimensional subspaces of X, Y, respectively, such that $z \in E \otimes F\}$ and the cofinite hull $\overleftarrow{\alpha}(z; X, Y) = \sup\{\alpha(\tilde{z}; X/K, Y/L) \colon K, L$ are finite-codimensional subspaces of X, Y, respectively$\}$. Here, $\tilde{z} = Q^X_K \otimes Q^Y_L(z)$, and Q^X_K, Q^Y_L are the canonical quotient maps. In general, $\overleftarrow{\alpha} \leq \alpha \leq \overrightarrow{\alpha}$. α is said to be totally accessible if $\overleftarrow{\alpha} = \overrightarrow{\alpha}$. The injective norm ε is totally accessible, but the projective norm π is not.

It is known that finitely generated tensor norms α are in one-to-one correspondence to maximal Banach operator ideals $(\mathfrak{A}, \mathbf{A})$ in the sense that one has an isometric isomorphism $\mathfrak{A}(E, F) = E' \otimes_\alpha F$ for all finite-dimensional Banach spaces E, F. In particular, α is totally accessible if and only if $\mathfrak{A}$ is totally accessible. The latter condition is equivalent to the condition that the adjoint and the conjugate ideals of $\mathfrak{A}$ coincide, that is, $\mathfrak{A}^* = \mathfrak{A}^\triangle$ isometrically. After introducing the so-called principle of $\mathfrak{A}$-local reflexivity, establishing several identities about adjoint, conjugate and dual operator ideals, and observing their relations to the accessibility of $\mathfrak{A}$, the author shows that the ideal $\mathfrak{L}_\infty$ of L_∞-factorizable operators and the ideal $\mathfrak{L}_1$ of L_1-factorizable operators are not totally accessible. Thus the associated tensor norms $g_\infty \sim \mathfrak{L}_\infty$ and $w_1 \sim \mathfrak{L}_1$ are not totally accessible. This answers an open problem stated in the book of A. Defant and K. Floret [*Tensor norms and operator ideals*, North-Holland, Amsterdam, 1993; MR1209438 (94e:46130)(p. 285)].

Reviewed by *Ngai Ching Wong*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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