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On normed products of operator ideals which contain $\mathfrak{L}_2$ as a factor. (English summary)
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Let $0 < p, q, r \leq 1$ be such that $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$. If $[\mathcal{A}, \alpha]$ is a p-Banach ideal and $[\mathcal{B}, \beta]$ is a q-Banach ideal, then the product ideal $[\mathcal{A} \circ \mathcal{B}, \alpha \circ \beta]$ is an r-Banach ideal (in the sense of A. Pietsch [*Operator ideals*, Translated from German by the author, North-Holland, Amsterdam, 1980; MR0582655 (81j:47001)]). If α and β are both the operator norm, then so is $\alpha \circ \beta$. But in general, $\alpha \circ \beta$ is not a norm even if α and β are. A typical example is $[\Pi_2 \circ \Pi_2, \pi_2 \circ \pi_2]$ where $[\Pi_2, \pi_2]$ is the Banach ideal of 2-summing operators: $\pi_2 \circ \pi_2$ is not equivalent to a norm. On the other side, $[\Pi_2^{\rm dual} \circ \Pi_2, \pi_2^{\rm dual} \circ \pi_2]$ (see Pietsch's book) is a Banach ideal, namely precisely the trace dual of the Banach ideal $[\Gamma_2, \gamma_2]$ of all Banach space operators which admit a factorization through some Hilbert space.

The author discusses several aspects of this problem and related topics and proves that if $[\mathcal{B}, \beta]$ is a maximal Banach ideal which is contained in the ideal $[\Gamma_\infty, \gamma_\infty]$ of $\mathcal{L}_\infty$-factorable operators, then $\mathcal{B} \circ \Gamma_\infty$ cannot be a Banach ideal.

Reviewed by *H. Jarchow*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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