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Conjugate operator ideals and the $\mathfrak{A}$- local reflexivity principle. (Konjugierte Operatorenideale und das $\mathfrak{A}$-lokale Reflexivitätsprinzip.) (German)

Kaiserslautern: Univ. Kaiserslautern, FB Math. 99 S. (1991).

One of the central questions when creating a theory of norms on tensor products of Banach spaces and/or a theory of operator ideals was how to define adjoint/conjugate tensor norms and operator ideals for infinite dimensional Banach spaces. In other words: which are the tensor norms/ideal norms which are "naturally" produced by the trace duality $\langle S,T\rangle :=$ trace $T\circ S$ (where $S\in\mathfrak{F}(E,F)=E'\otimes F$ and $T\in\mathfrak{F}(F,E)=F'\otimes E$ are finite rank operators), since the trace is naturally defined on the completed projective tensor product $E'\widetilde{\otimes}_\pi E$ and, a priori, not on any complete operator ideal (in particular not on the nuclear operators) it is obvious that tensor product methods help understanding the trace duality for operator ideals. Schatten's approach in the fourties worked, more or less, only on Hilbert spaces since there (by reflexivity and excellent approximation properties) all "reasonable" definitions coincide. It was *A. Grothendieck* in his "Résumé de la théorie métrique des produits tensoriels topologiques" in 1954 who proposed the approach via a canonical extension from finite dimensional subspaces (where everything runs well) to arbitrary Banach spaces. Later on, in 1971, Pietsch defined the adjoint operator ideal $\mathfrak{A}^*$ — his definition is natural under the point of view of Grothendieck's theory of tensornorms; Gordon-Lewis-Retherford introduced in 1973 a related "conjugate" operator ideal $\mathfrak{A}^\Delta$, which — in the contrary — is somehow "skew" under this point of view but nevertheless quite useful. It is easy to see that the various approximation properties, the accessibility of tensor norms (a notion due to Grothendieck 1954) and the accessibility of operator ideals (A. Defant 1986) are intrinsically related to a "good behaviour" of trace duality.

The dissertation under review gives many results in this direction. It studies carefully the relation between the adjoint and conjugate operator ideals (by using adequate tensor product descriptions), gives various quotient formulas in this context, shows that accessibility is somehow even necessary for the good behaviour of $\mathfrak{A}^*$ and $\mathfrak{A}$ on the class of all Banach spaces, introduces a weak principle of local reflexivity for operator ideals, obtains — with this — results about $\mathfrak{A}(M,F'')=(\mathfrak{A}(M,F))''$ to hold (M a finite dimensional and F an arbitrary Banach space) and gives many observations about the accessibility of tensor norms and operator ideals. This latter investigation was made also under the objective to facilitate the search for a non-accessible finitely generated tensor norm (which is the same as a non-accessible maximal normed operator ideal). After the dissertation has been finished, Pisier constructed such a norm [see the forthcoming book "Tensor norms and operator ideals" by *A. Defant* and the reviewer]thus solving another important problem from Grothendieck's résumé to the negative.

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Keywords : norms on tensor products of Banach spaces; operator ideals; conjugate tensor norms; ideal norms; trace duality; adjoint operator ideal; approximation properties; accessibility of tensor norms; accessibility of operator ideals; relation between the

adjoint and conjugate operator ideals; quotient formulas; weak principle of local reflexivity for operator ideals; non-accessible finitely generated tensor norm; non- accessible maximal normed operator ideal
Classification :
∗46B28 Normed linear spaces of linear operators, etc.
47L20 Operator ideals
46M05 Tensor products of topological linear spaces
46A32 Topological linear spaces of linear operators, etc.